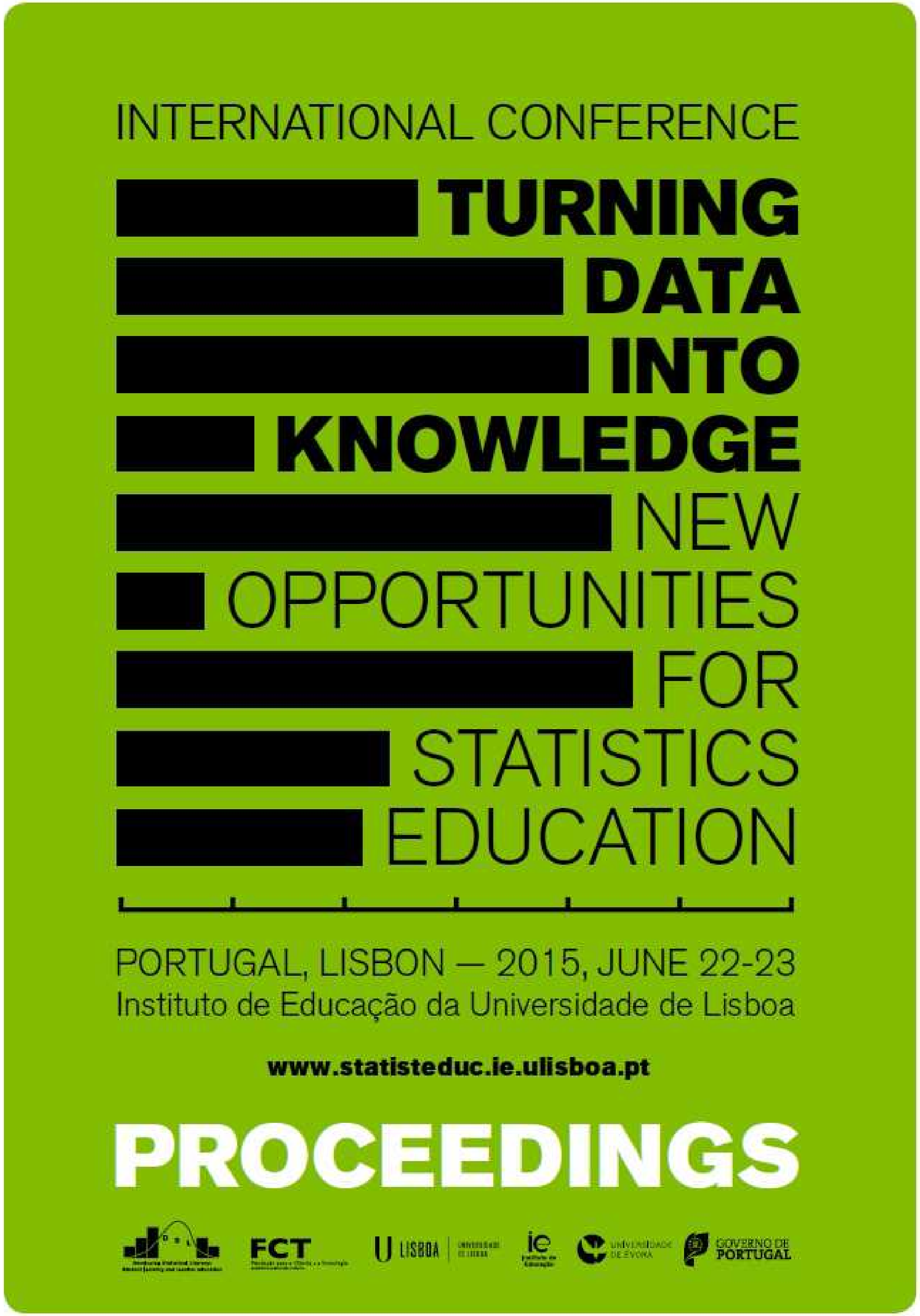

PROCEEDINGS OF THE INTERNATIONAL CONFERENCE TURNING DATA INTO KNOWLEDGE: NEW OPPORTUNITIES FOR STATISTICS EDUCATION

22-23 JUNE, 2015

Institute of Education of the University of Lisbon

Portugal

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Proceedings of the International Conference Turning data into knowledge: New opportunities for statistics education

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CONTENTS

INTRODUCTION

WELCOME TO LISBON, WELCOME TO THE CONFERENCE

"Over seven hills, which are as many points of observation whence the most magnificent panoramas may be enjoyed, the vast irregular and many-coloured mass of houses that constitute Lisbon is scattered. For the traveler who comes in from the sea, Lisbon, even from afar, rises like a fair vision in a dream, clear-cut against a bright blue sky which the sun gladdens with its gold. And the domes, the monuments, the old castles jut up above the mass of houses, like far-off heralds of this delightful seat, of this blessed region. The tourist's wonder begins when the ship approaches the bar, and, after passing the Bugio lighthouse – that little guardian-tower at the mouth of the river built three centuries ago on the plan of Friar João Turriano –, the castled Torre de Belém appears, a magnificent specimen of sixteenth century military architecture, in the Romantic-Gothic-Moorish style. As the ship moves forward the river grows narrower, soon to widen again, forming one of the largest natural harbours in the world with ample anchorage for the greatest of fleets. Then, on the left, the masses of houses cluster brightly over the hills. That is Lisbon.

Fernando Pessoa in "Lisbon, what the tourist should see" (1925) http://lisbon.pessoa.free.fr/InteractiveBook.php

Lisbon's description by the famous Portuguese poet Fernando Pessoa introduces you to some of the many beautiful types of scenery in this city, with the almost constant presence of sunshine and the River Tagus. It is a city whose history spans back thousands of years, which you can discover walking through its seven hills. There are innumerous experiences that can be lived in Lisbon: walking through characteristic neighborhoods, visiting streets filled with heritage monuments and museums, relaxing in one of the many gardens, belvederes and esplanades, and enjoying the pleasure of its gastronomy.

We are delighted to welcome you to Lisbon and particularly to the International Conference *Turning data into knowledge: New opportunities for statistics education*, being held at the University of Lisbon. This public university has eighteen faculties and institutes throughout the city of Lisbon and surrounding municipalities. It was the first university created in Portugal, in 1288. The university was later transferred to the Portuguese city of Coimbra, in 1537. In the late eighteenth century, it was reestablished in Lisbon. Recently, in 2013, the University of Lisbon merged with the Technical University of Lisbon, forming the ULisboa that is now the largest university in Portugal with more than 48 000 students.

This conference has been an initiative of the Portuguese research project DSL – *Developing statistical literacy: Student learning and teacher education* – involving the Institute of Education of the University of Lisbon and the University of Évora,

and supported by a group of leading international researchers in statistics education who took part in the Scientific Committee to whom we are deeply grateful.

The Conference was sought to be an opportunity to gather researchers, teacher educators and teachers interested in statistics education, to exchange experiences, and to present and discuss recent research and current projects. Therefore we invite all participants to contribute actively to the debate throughout the conference sessions and to meet different people at the lunch and coffee breaks to share ideas and experiences.

The Scientific Programme

Currently, statistics education takes place in a new social and cultural context and faces a global challenge of meeting the calls for statistically literate and informed citizens who are able to **turn data into knowledge**. Such a challenge provides new opportunities to rethink both what statistics we teach and how we teach statistics. Doing so is imperative in order to develop students' ability to reason about data and to use them effective and critically, in their daily life, for prediction and decision-making.

It is well known by both researchers and practitioners in school that statistical literacy and reasoning, which call for critical, investigative and communicative skills, prove to be difficult to students. Thus, current international curriculum recommendations suggest data-orientated approaches for teaching statistics, at all levels of schooling, using real data and taking advantage of available technology. Furthermore, to develop their statistical literacy and reasoning, students are expected to deal with data in significant and authentic contexts. This conference addresses these challenges and opportunities for statistics education, for all school levels.

Two strands have been defined for the conference – Statistical literacy and Statistical reasoning – and these are the broad themes of the two Plenary Talks, presented by Janet Ainley and Dani Ben-Zvi, respectively. These two strands encompass a diversity of topics of research and projects, such as technology-enhanced learning and teaching practice with technology, as well as research in teacher education and in professional development, namely teachers' systematic research about their own practice. More broadly the programme also intends to add to the discussion about statistics education, namely in what concerns its aims and diverse curriculum issues through the Plenary Panel that will take place in the last part of the conference.

This 2-day Conference includes also other types of sessions: Research Papers, Short Oral Communications, Posters presentations and Workshops. All the received contributions as Research Papers (RP), Short Oral Communications (SO) and Posters were submitted to a blind review process and the final contributions are now collected on these proceedings. We would like to thank the reviewers, whose names you can find at the final section of these proceedings, for their valuable contribution to the scientific quality of the conference.

The organization received originally 12 RP, 19 SO and 14 Posters. Through the review process some of these have been suggested to be resubmitted as SO or Poster. Unfortunately, a few colleagues whose proposals had been accepted did not obtain the support they needed to attend the conference. Finally, we have a total of 8 RP, 19 SO and 14 Posters in these proceedings that will be presented in the conference. The scope and diversity of the proposals' themes are a clear evidence of the richness of the field of statistics education, nowadays.

Additionally, two workshops in Portuguese also will take place during the conference, targeting teachers who teach in elementary and secondary levels. We are thankful to Cláudia Oliveira, Marisa Gregório (Workshop 1 – *Promoting statistical reasoning with the TinkerPlots*), Rosa Tomás Ferreira and Sandra Quintas (Workshop 2 – *Promoting statistical investigations in the classroom*) who were responsible for organizing and conducting the workshops in connection with Ana Henriques and myself. More than 30 Portuguese teachers and teacher educators have registered to attend these two workshops.

The conference also provides the opportunity to share part of the work that has been developed by the project DSL, and that is presented in a Plenary Panel, as well as in different RP, SO and Posters by several of its members. We thank you for coming and hope you will enjoy Lisbon and the conference.

Hélia Oliveira, Conference co-chair

PLENARY TALKS

THE USES OF STATISTICAL LITERACY

Janet Ainley

University of Leicester, United Kingdom

ABSTRACT

When young children learn to read and write in their mother tongue, which is the normal meaning of 'literacy', they already know what these skills are useful for. They experience adults using reading and writing for a range of purposes everyday, and as their skills develop they can immediately use reading and writing for many of the same purposes as adults. They can read text in books, magazines, birthday cards, on computer screens, and write lists, text messages and notes to friends. The uses and purposes of statistical literacy are much less visible to children, and indeed to many adults. Even when statistical concepts and skills, such as sampling, or measures of average, are taught, the power and usefulness of those ideas is not always clearly connected to the experiences of learners. As a result, students and adults may 'understand' different measures of average, but not know why and when such as measure might be useful or appropriate. In this talk I will argue for the need for pedagogic approaches which foreground the uses of statistical ideas in ways which are meaningful for learners, and thus contribute to the development of useable statistical literacy.

THE CHALLENGE OF DEVELOPING STUDENTS' STATISTICAL REASONING

Dani Ben-Zvi

University of Haifa, Israel

ABSTRACT

Statistical reasoning is the way people reason with statistical ideas and make sense of statistical information. This involves making interpretations based on sets of data, graphical representations, and statistical summaries. Statistical reasoning combines complex ideas about data and chance, which leads to making inferences and interpreting statistical results. Underlying statistical reasoning is a conceptual understanding of a web of important ideas, such as distribution, center, variability, association, uncertainty, randomness, and sampling. In this presentation, I reflect on research projects – conducted during the past 20 years with colleagues and graduate students – on students' learning to reason statistically. I contemplate on what I have learnt and offer some lessons. The questions I discuss are: What is statistical reasoning? Why is it challenging for learners at all ages? What can research tell us about students' statistical reasoning and its development? How can we support students' statistical reasoning? What research challenges are of current interest to the statistics education community?

PLENARY PANEL

STATISTICS EDUCATION: ISSUES AND PERSPECTIVES

Janet Ainley[1], Dani Ben-Zvi[2], Andreas Eichler[3], João Pedro da Ponte (Chair)[4]

[1]University of Leicester, United Kingdom

[2]University of Haifa, Israel

[3]University of Kassel, Germany

[4]University of Lisbon, Portugal

This panel addresses the following issues:

a) Aims: What must be the aims of statistics education in primary, middle and secondary schools? What are the main obstacles to achieve them?

b) Curriculum issues: What is reasonable to strive for in primary education? A privileged relationship with mathematics or a privileged relationship with other topics?

c) Research and Practice: What may be possible contributions of researchers to improve the status of statistics education in primary, middle and secondary schools?

PROJECT DSL PANEL

PROJECT DEVELOPING STATISTICAL LITERACY: STUDENT LEARNING AND TEACHER EDUCATION

Hélia Oliveira[1], Ana Paula Canavarro[2], Ana Henriques[1], Susana Colaço (Chair)[3]

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ABSTRACT

Recent international and national mathematics curriculum guidelines indicate the development of students' statistical literacy, at different levels of schooling, as a major educational aim. In Portugal, the mathematics syllabus for basic education, which began to be implemented in 2009, gave a greater emphasis on statistics, presenting more demanding learning goals, since the elementary levels. This represented a challenging situation for practicing teachers, requiring them to develop new perspectives about the teaching and learning of statistics. In this context, the project *Developing statistical literacy: Student learning and teacher education* was planned aiming to study the development of statistical literacy from elementary to secondary education, with special attention to two main issues: i) the characterization of key aspects of students' statistical literacy, particularly regarding the ability to formulate questions, collect data and represent them to answer those questions, and ii) the development of statistical and didactical knowledge for teaching in different schools levels. As the project unfolded, another research strand associated with students' statistical reasoning and the necessary conditions for its development emerged.

Regarding the development of statistical literacy and reasoning, the project assumes the key role of students doing statistical investigations, as this activity tends to engage them in meaningful learning and promotes a holistic view of statistics, mobilizing knowledge in various areas, using complex reasoning processes in a flexible way and helping them to develop critical thinking. In particular, in what concerns statistical knowledge development, the project has been focusing, among others, on the theme of statistics informal inference through the design of learning environments with technological resources that support students' reasoning processes.

The project has also focused on deepening the field of teachers' statistical and didactic knowledge for teaching statistics and their development in the context of pre-service or in-service teacher education. The knowledge gained by the project research team has supported the planning of courses in teacher education for prospective teachers of different grade levels, as well as of professional development courses for teachers supported by collaborative contexts that consider the articulation between theory and practice.

The research methodologies adopted by the project team have been diversified. There is a strong incidence of qualitative and design research approaches, both for studying students' statistical literacy and reasoning, and teacher education scenarios and teachers' practice.

The project is developed under the responsibility of the Institute of Education of the University of Lisbon, in partnership with the University of Évora. The team comprises seven senior researchers from different national higher education institutions, with extensive experience in teacher education. The team also includes PhD and master's students, who are teachers or teacher educators in various parts of the country. The results and products of the project have been disseminated in national and international contexts, through the publication of papers and communications in scientific and professional meetings, and in seminars opened to the national and international mathematics and statistics education communities.

The project had the financial support of FCT - Fundação para a Ciência e Tecnologia, from 2012 to 2015 (contract PTDC/CPECED/117933/2010).

THE PROJECT TEAM

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Other members and collaborators

Ana Caseiro Rodrigues, Ana Isabel Mota, Cátia Freitas, Cláudia Fernandes, Cristina Roque, Isabel Velez, Marisa Gregório, Mónica Patrício, Nélida Filipe, Patrícia Antunes, Paula Gil, Raquel Santos, Sandra Cadima, Sandra Quintas.

RESEARCH PAPERS

UNDERSTANDING CONDITIONAL PROBABILITY THROUGH VISUALISATION

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[2]University Kassel, Germany

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This paper refers to a research project aiming to compare two different strategies to visualise conditional probabilities for which learning obstacles are often reported. We firstly explain the mentioned two strategies, i.e. the tree diagram with natural frequencies and the unit square. Afterwards we provide the development of our test instrument and discuss some results referring two samples of prospective teachers. These results show considerable differences between the two visualisations as well as a significant benefit of the unit square.

INTRODUCTION

"Representation and visualization are at the core of understanding in mathematics" (Duval, 2002, p. 312). In both mathematics education and statistics education, it is widely accepted that visualisation could have a considerable impact on students learning of mathematics or statistics. This is even the case, if we restrict the term visualisation to graphical representation (c.f. Presmeg, 2006). However, research in mathematics education, statistics education and cognitive psychology gave evidence that visualisation does not necessarily foster students' understanding. Thus, the efficiency of visualisation is dependent on the quality of a specific visualisation as well as on the quality of the connection of a certain visualisation and a mathematical object and thus, the expectable outcome for enhancing students´ understanding (e.g., Ainsworth, 2006; Presmag, 2006). For this reason, a crucial question in research in mathematics education and statistics education is to identify the one of potentially different visualisations that is most efficient referring to students' learning. A second question that is closely connected to the first question concerns the definition of that part of students' knowledge that should be promoted by certain visualisation.

The aim of this research, which is a part of the German research project VisDeM (visualisation in teaching German language and mathematics, http://www.kebu-freiburg.de/visdem/), is to gain results referring to the two questions outlined above. Regarding these two questions, we focus on the subject of conditional probability, and, particularly, on the subject of the Bayes' rule. The rational for this focus is twofold. On the one side conditional probabilities are a subject in which the human intuition often seems to be misleading (e.g., Kahneman & Tversky, 1972). On the other side research gained strong evidence for the efficiency of two different forms of visualisation for the learning of conditional probabilities and also the Bayes' rule, i.e. a tree with natural frequencies (e.g., Sedlmaier & Gigerenzer, 2001) and the unit square (Bea, 1995). However, the aforementioned research results do neither involve

a direct comparison referring to the impact of these both visualisations on students' learning nor a direct comparison by only using natural frequencies.

In this paper, we firstly discuss the tree diagram, the unit square and research related to these two visualisation strategies. Afterwards we address the two research questions outlined above in two subsequent steps referring to two different samples of prospective teachers. To each step, we discuss the method, specific research questions and the results. We conclude this paper by giving a brief explanation of further steps of our research.

VISUALISATION REFERRING TO CONDITIONAL PROBABILITIES

We show both visualisations (the tree diagram with natural frequencies and the unit square) in figure 1 referring to the following situation:

- 10 % of a certain population has a disease without knowing that.
- A diagnosis test gives a correct indication of the disease in 80 % of all cases (i.e. the persons that have actually the disease).
- The diagnosis test has further the characteristic that 10 % of cases will be wrongly identified as having the disease, although the disease is not present.

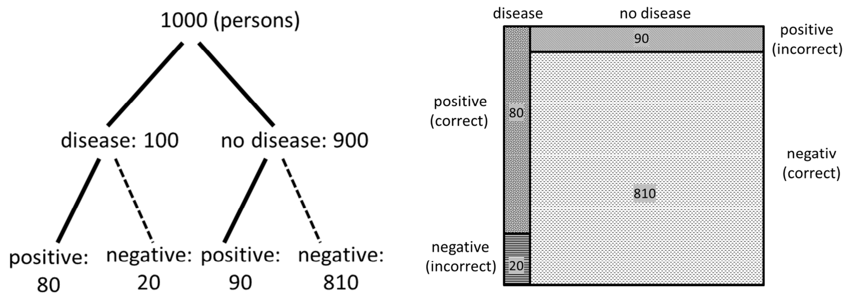

Figure 1: Tree diagram (left side) and unit square (right side) referring to the same situation of a diagnosis test.

The tree diagram with natural frequencies has a sequential and hierarchical structure. Thus, for example, the very right path in the tree diagram represents the persons with no disease who got a negative result in the diagnosis test. The unit square is a statistical graph where the persons with no disease, who got a negative result in the diagnosis test, are represented by a partial area of the square, in this case the bottom right one (c.f. Eichler & Vogel, 2010).

Sedlmeier and Gigerenzer (2001) investigated adults (students at university) concerning their ability to solve tasks using the Bayes' rule. They found that the tree diagram with natural frequencies is more efficient than a common tree diagram with probabilities. Particularly, the learning results based on the intervention involving the tree diagram with natural frequencies seem to be stable on a high level over a long period. In contrast the high immediate learning of students based on the intervention with the tree diagram with probabilities showed a strong decay after few weeks. Wassner (2004) replicated these results referring to students' learning in schools using a qualitative design.

Sedlmeier and Gigerenzer (2001) gave two reasons explaining the efficiency of the tree diagram with natural frequencies. The first reason concerns the way of information‘s representation based on natural frequencies (c.f. Gigerenzer & Hoffrage, 1995). The second reason refers to the assumption that the tree represents a powerful tool to structure given information. In the research of Sedlmaier and Gigerenzer (2001), the efficiency of students' learning was measured by the number of students' correct solutions of tasks for which applying the Bayes' rule is necessary. Thus, this study primarily addressed students' procedural knowledge (Hiebert & Carpenter, 1992), i.e. the knowledge how to solve a specific class of tasks.

Further, Bea (1995) yielded evidence that using a unit square is more efficient than using the tree diagram with probabilities when students' learning of conditional probability and the Bayes' rule was regarded. In his research, Bea used the unit square with probabilities. According to the results of Gigerenzer and Hoffrage (1995) mentioned above we also give the information in the unit square as natural frequencies (fig. 1). As mentioned before, the unit square visualises conditional probabilities and Bayes' rule geometrically based on proportions of line segments and proportions of partial areas of the square. Accordingly, Bea (1995) showed that the unit square particularly had promoted students' conceptual knowledge (Hiebert & Carpenter, 1992). Conceptual knowledge in this field concerns, for example, the knowledge how resulting probabilities would change if probabilities that determine the shape of the unit square (e.g., the probability of having a disease) would be changed.

Based on the considerations above, our research does not only focus on a comparison of the general efficiency of the two strategies of visualisation, but also focuses on the efficiency of the strategies of visualisation to promote primarily procedural knowledge or primarily conceptual knowledge.

FIRST STEP OF RESEARCH

Method

Although there is research about the efficiency of both strategies of visualising conditional probabilities, only the tree diagram with natural frequencies plays a substantial role in statistics education in Germany, where our research took place (Eichler & Vogel, 2010). For this reason, one specific aim of the first step of our research approach was to examine if there are differences in extracting the relevant information from the two visualisations. To describe different levels of extracting information from a visualisation we used the model of Curcio (1987; 1989), referring to the aspects *read the data*, *read between the data*, and *read beyond the data*. We explain these aspects below when exemplarily discussing the test items.

The sample consists of 78 prospective mathematics teachers at the University of Kassel that were enrolled in a course of mathematics education in 2014. This sample was randomly divided into two subsamples.

The participants got a brief description of a visualisation based on a simple example, the first subsample referring to the use of the tree diagram with natural frequencies, the second sample referring to the unit square with natural frequencies. Both descriptions were identical as regards stochastic content, they differ only as regards the kind of visualisation.

We integrated the three aspects of Curcio (ibid.) that we describe above in test items. The aspect *read the data* is described as lifting information to answer explicit questions for which the obvious answer is right there in the graph (Curcio, 1989). An example of the aspect *read the data* is shown in figure 2 referring the context of diseases.

Read the numbers from the diagram:

a) ____ have the disease got a positive test.

b) ____ have not the disease.

c) ____ of all persons got a negative test.

d) ____ are negatively tested persons that have not the disease.

e) ____ of all persons got a positive test.

Figure 2: Example of an item referring to *read the data*.

We used two items with other contexts for testing the prospective teachers' ability to read the data. Both items consist of five tasks that are equivalent to the example shown in figure 1 and 2.

The aspect *read between the data* is described as interpolating and finding relationships in the data presented in a graph (Curcio, 1989). An example of the aspect of *read between the data* is shown in figure 3.

Again, we used two items with other contexts for testing the prospective teachers' ability to read between the data (speaking dialect and doing physics). Both items consist of five tasks that are equivalent to the example shown in figure 1 and 3.

Mark whether the following assertions are true or false:

a) Most of the persons having the disease got a positive test (true/false).

b) Most of the persons have not the disease (true/false).

c) Most of the persons, who got a positive test, have the disease (true/false).

d) Most of the persons, who got a negative test, have not the disease (true/false).

e) Most of the persons, who got the disease, got a negative test (true/false).

Figure 3: Example of an item referring to *read between the data*.

Finally, Curcio (1989) described the aspect of reading beyond the data as the ability of extrapolating, predicting and inferring from the visualisation to answer implicit questions. One example referring this aspect was the situation shown in figure 1 with the task shown in figure 4.

How is the change of the following proportions if the percentage of persons that have the disease would be bigger? Mark the correct solution.

a) The percentage of persons having the disease that get a positive test result will be bigger/smaller/constant.

b) The percentage of persons getting a positive test result that have the disease will be bigger/smaller/constant.

c) The percentage of persons that get a negative test will be bigger/smaller/constant.

d) The percentage of persons having the disease that get a negative test result will be bigger/smaller/constant.

e) The percentage of persons getting a negative test result, that have not the disease will be bigger/smaller/constant.

Figure 4: Example of an item referring to *read beyond the data.*

We used this and one further item with another context (concerning flower cultivation) for testing the prospective teachers' ability to read beyond the data.

Each sub-item was coded with 0 (false) or 1 (correct). We had two hypotheses for this first step of our research:

- Because the requested information is directly given within the tree diagram as well as within the unit square we expect no difference in students' solutions with regard to Curcio's (1989) levels of reading data and reading within the data. Thus, we hypothesize that there is no difference between the two strategies of visualise conditional probabilities referring to the ability to read the data and to read between the data.
- Because the proportions are (at least partially) displayed within in the unit square but not within the tree diagram (beyond the same numerical information given within both kinds of visualisation) we expect the unit square to be more helpful for concluding new information from the given on. This applies to the level of reading beyond the data. Thus, we hypothesize that the unit square is more appropriate for reading beyond the data.

Results

The item tasks referring to the aspect of *reading the data* seems mostly to measure the same construct (for ten items Cronbach's alpha is 0.616). Actually, the results shown in figure 5 imply that there is no reason to reject our first hypothesis. The

items show at most a very high rate of correct answers. All existing differences are far away from being significant.

These results are important for our further research steps: Although the strategies of visualising conditional probabilities are different referring to their degree of familiarity in school, the ability of the students of extracting information from the diagrams do not differ. Thus, it is possible to directly compare the results of the other tasks.

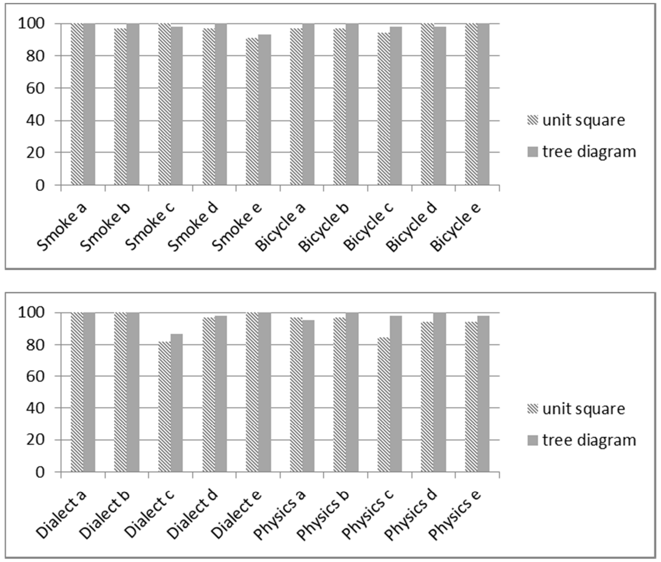

Figure 5: Results referring to the items concerning read in the data (smoke and bicycle) and read between the data (dialect and physics).

The results referring to the third aspect of Curcio's model, i.e. read beyond the data is shown in figure 6.

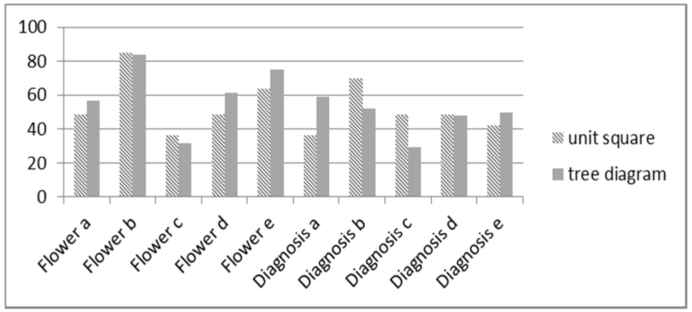

Figure 6: Results referring to the items concerning read beyond data.

Actually, most of the single items and especially the sum of the item scores (for ten items Cronbach's alpha is 0.616) do not show a significant difference between the two visualisations. Thus, our second hypothesis could not be rejected.

However, in a further exploratory investigation of the data, we found that the results differ in case that the years of study of the prospective teachers are regarded. It means, if our sample is restricted to those students that are not in the first year of their studies, the results of our test are different from those referring to all the prospective teachers. Referring to this restriction we have 39 cases of which 21 worked with the tree diagram and 18 cases which worked with the unit square. Here, the reliability of the aspect read beyond the data indicated by Cronbach's alpha is 0.649 and, thus, slightly better than regarding all cases. Further, the difference between the performances of the prospective teachers compared by the two visualisations is in fact not significant, but the p-value implied that the difference between the two visualisation strategies could become significant if particularly experienced prospective teachers are regarded. Among other things, this informal and explorative result was the basis for the second step of our research.

SECOND STEP OF RESEARCH

Method

The sample in this step of our research consists of 206 prospective teachers at the University of Education Heidelberg. 58 of these prospective teachers were in their first years of study the other 148 prospective teachers had more experience, but not with regard to systematic learning neither concerning conditional probabilities and Bayes' rule nor concerning these two kinds of visualisations on which we focus.

Again the sample was randomly divided into two subsamples. Each of the subsamples got three test items referring to the aspect of read beyond the data. Each of the items consists of five sub-items. The tests referring to the two subsamples differed in terms of the given visualisation of the addressed situation. Further, each subsample got the same brief introduction to the used visualisations that was the same as in the first step of our research.

One of our hypotheses for this step of research was that there is a difference between the ability to read beyond the data referring to the different strategies of visualise conditional probabilities. However in this case, we particularly expected a significant result referring to experienced prospective teachers, because we expected them to be able to use the graphical advantage of the unit square (cf. reasoning of hypothesis 2 in the first step of research) more effectively for reason of their further developed general mathematical abilities.

Results

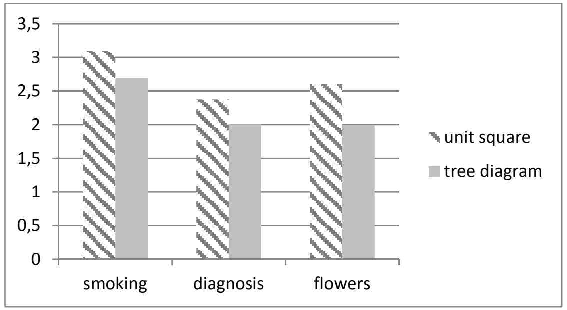

Figure 7: Results referring to the items concerning read beyond data.

Actually, if the whole group of the prospective teachers is regarded, there is no significant advantage of the unit square compared to the tree diagram ($p = 0.126$). However, if only the experienced prospective teachers are regarded (years of study is more than one year), there is statistical evidence that the unit square is more efficient than the tree diagram ($p = 0.042$).

In figure 7 we provide the results of the sum of the each five sub-items that generally show an appropriate reliability (Cronbach's alpha is 0.75). These results refer to the average of correct answers to the five sub-items of the three items.

DISCUSSION AND CONCLUSION

In this paper we reported a study consisting of two steps aiming to compare two strategies of visualise conditional probabilities, i.e. the tree diagram with natural frequencies and the unit square.

We firstly investigated whether there is a difference between the two visualisations referring to extracting simple information from the diagrams that could be assigned to the aspects of reading the data and reading between the data proposed by Curcio (1989). As theoretically assumed we found no empirical evidence for such a difference between the two visualisations. This absence of a difference was an important result for further research. Actually the tree diagram with natural frequencies is often promoted as an outstanding way to visualise conditional probabilities (e.g., Gigerenzer & Hoffrage, 1995; Sedlmeier & Gigerenzer, 2001; Spiegelhalter & Gage, 2014). However, our research gives evidence that there is another visualisation that seems to be equally efficient to extract relevant information of a statistical situation.

Moreover, for experienced prospective teachers, our results further imply that the unit square is more efficient to solve tasks referring to the aspect of reading beyond the data or rather to solve tasks that focus on a conceptual understanding of conditional probabilities and the rule of Bayes. This was also an important result that led to the hypothesis that a higher mathematical education increases the benefit of the unit square compared to the tree diagram with natural frequencies. Following this paper, the mentioned hypothesis refers to the aspect of reading beyond the data. For this reason, we plan further steps of our research. For example, we want to investigate differences among samples of students of sciences, samples of students of mathematics and samples of students that have no relation to mathematics. Further we plan to include a clearer distinction between items that address procedural knowledge and conceptual knowledge for proving how different visualisations impact on different aspect of knowledge. One aim of these further steps of our research is to precisely identify situations of uncertainty for which the unit square is more efficient for students learning than the tree diagram or other strategies of visualising conditional probabilities.

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PROBABILITY CONTENT IN THE ENTRANCE TO UNIVERSITY TESTS IN ANDALUCIA

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In this work we analyse the contents of the probability problems proposed for the university entrance tests in Andalucía (Spain). Specifically, we examine the problems proposed to the students who selected the specialty of Social Sciences. We solved all the probability problems for each of the 6 different tests proposed in 2003, 2008 and 2013, and using semiotic analysis we identified the probabilistic objects involved in their solution. An elementary statistical study of the distribution of the different objects in the three years served to obtain conclusions about the presence of the objects in the problem and about the difficulty of the problems.

INTRODUCTION

The need for probability education has been recognized in the Spanish curricular guidelines, since probability contents were included in Primary and Secondary Education (Batanero, Arteaga, & Gea, 2011; Batanero, Gea, Arteaga, & Contreras, 2014). Probability is also included in Mathematics into the specialties of Sciences and Technology, and Humanities and Social Sciences in High School (MEC, 2007), where we find the following contents:

- Assigning probabilities to events. Prior and posterior probability; joint, conditional and total probability, Bayes' theorem.
- Binomial and Normal distributions. Practical implications of central limit theorem; approximation of binomial distribution by the normal distribution; law of large numbers.

Assessment is an important element that guides the teacher and the students towards achieving the learning goals. Assessment is given a main role in the transition from high school to university in Spain, since the score the students get in the compulsory university entrance tests, often determines the student's possibility to follow their preferred university degree (and future career). Paradoxically, probability problems are only included in the tests aimed at Humanities and Social Sciences students, although this content is also taught to Science and Technology students.

In this paper we present preliminary results of a project directed to analyse the content of the university entrance tests in Andalucía. The pertinence of this project is clear, since these tests often influence (more than the curricular guidelines) what is taught to these students, and our results may help the teachers to prepare their students for these tests. These preliminary results consist in the analysis of all the problems posed in three different years (36 problems in total) and will help to try the method and determine the main variables to continue the analysis of all the test

proposed in the last 14 years. After summarizing the theoretical framework and related research we describe the method. Then, we include the detailed analysis of one of the problems included in these tests. Finally, we present a statistical summary of the results obtained in the tests analysed and conclude with some implications for teaching.

THEORETICAL FRAMEWORK

We base on the onto-semiotic approach to mathematics education (Godino, & Batanero, 1994; 1998: Godino, Batanero, & Font, 2007) where meaning is modelled in terms of systems of the practices carried out when solving a given problem. In these practices different type of objects intervene; "object" is understood in a broad sense and includes problem-situations; procedures; concepts and properties. The mathematical practices can be idiosyncratic of a person (personal meaning) or shared within an institution (institutional meaning). In mathematics education research we are interested in the meaning of specific objects in teaching institutions; in particular in this research we are interested in analysing the institutional meaning of probability in the entrance tests to University. This meaning can be described by the mathematical objects linked to the problems proposed in these tests and the practices that students should carry out in their solution. The method of semiotic analysis proposed in this framework is directed to identify these objects and the interpretive processes needed by the person who solves the problem. When this analysis is applied to the problems included in a textbook or in a test it serves to identify the correspondence between assessment and curricular guidelines and to show the complex work required from the student.

PREVIOUS RESEARCH

We also base on two previous research that has analysed probability problems in textbooks with the same framework, to identify the variables used in our study. Ortiz (1999) analysed a sample of high school textbooks and identified as main concepts implicit in the problems those of compound experiments, dependence and independence, joint probability, total probability and the Bayes theorem. In our analysis we will consider all these objects. According to him the textbooks problems were consistent with the curricular guidelines for high school in Spain and few problems were related to conditional probability.

Díaz (2004) studied the probabilistic problems in 17 university textbooks and classified the problems according to the following variables: a) type of experiment: sampling with or without replacement, or simple experiment; b) dependent or independent experiments; c) property or theorem used: product rule, computation of conditional probability using a formula; total probability or Bayes' rule; d) type of situations: synchronic (when experiments are simultaneous) or diachronic (consecutive experiments). She did not informed of the frequency of the different problems.

Although these two authors identified variables and categories relevant in the analysis of probability problems, they limited themselves to identify the presence or absence of each category in the sample of textbooks they analysed. Their studies did not highlight the complexity of the problems, because they did not perform a semiotic analysis of the same. Moreover we found no previous analysis of the problems proposed in the entrance to university tests; consequently our paper is a first contribution to this research question and complements these previous studies.

METHOD

The university entrance tests in Andalucía are currently regulated by the Royal Decree 1892/2008, of 14 November, where the conditions for access to official university studies and the admission procedures to Spanish public universities are described (MP, 2008). This document requires the students to pass a maturity test (university entrance test), in which the students' knowledge and abilities are assessed.

The mathematics test for Humanities and Social Sciences students consists of two options: A and B. The student can chose one of these options and should solve all the problems proposed in the selected option (mixing problems from both options is not allowed). The content of tests (4 problems) is similar for both options: the first problem is related to algebra, the second to analysis, the third to probability and the last to statistical inference. The correct solution to each problem is scored with 2.5 points from a total of 10 points in the full test. The content of the problems should be in agreement with the curricular guideline contents for high schools in Spain.

In our study, we focus only on the probability content of this curriculum. As we mentioned above, each test includes a specific problem related to this content. Moreover, as we will show in our analysis this problem is centre specifically in "*Reinforcing ideas about prior and posterior probabilities, joint probability, conditional probability; total probability and the Bayes' rule*".

In this preliminary study we examine the probabilistic problems contained in the tests proposed in the years 2003, 2008 y 2013. Since we are interested in the period 2003 (when the current regulation for the entrance test were started) to 2014, we selected a year at the beginning, the middle and the end of the period, in order to see if there is an uniformity in the content of these tests in the different years or a tendency to change is visible. In each of these years, 6 different versions of the tests were used (depending on the semester and city); since each test has two options, we analysed a sample of 36 problems.

Given that the sample size is limited, our study is exploratory, which is usual in qualitative research. We therefore do not intend to extrapolate the results to other different tests. However, we believe that our findings can serve to formulate some preliminary hypotheses about the probability content of the tests in other years or in other regions that would be tested in the final study.

SEMIOTIC ANALYSIS OF PROBLEMS: AN EXAMPLE

Each of the items included in the sample was solved; and we performed a semiotic analysis of the correct solution to the problem, in order to identify the mathematical content required in the solution. In this section we present the analysis of item P5A included in the fifth version of the test (option A) in 2013 as an example.

Item 5A. From a total of 212 elderly people in a geriatric home, 44 of them have lung problems; 78 of them are regular smokers, and only 8 have lung disease and are non-smokers.

a. What is the probability that an elderly person in this home, chosen at random, does not smoke and does not have lung disease?

b. What percentage of elderly people with lung problems are smokers?

To correctly solve the problem, the student needs to identify the data of the problem. First, the student should consider the compound experiment *selecting an elderly person randomly* where it is necessary to identify two characteristics (two simple experiments): a) On one hand, the person may have lung disease or not (the sample space has two events: *having lung disease*, L, and *not having lung disease* L^C; b) secondly, the person may be a smoker or not (the sample space in the second experiment has two events: *smoking*, *S*, and *not smoking*, S^C). A two-way table (See Table 1) may help the student to identify the frequency of different categories of people in this home (see Table 1).

	L	L^C	TOTAL
S	36	42	78
S^C	8	126	134
TOTAL	44	168	212

Table 1: Data needed to solve problem 5A.

Since simple events include a single element and compound events at least two elementary events, we conclude that the events considered in the problem are simple events in a compound experiment. The student must identify each simple experiment and the compound experiment, that is, form the Cartesian product of two simple experiments. Once the events and frequencies are identified, the student should interpret the questions posed in terms of probability.

Solution to question a.

Table 2 shows a summary of the steps that the student must follow to solve this question. In the first question, the student has to interpret that in this question we ask to compute a joint probability; specifically the probability of the intersection between being a non-smoker and having no lung disease $P(S^C \cap L^C)$. To solve this first question, the student has to identify the joint frequency corresponding to the intersection of these events in Table 1 and divide this frequency by the total of elderly people in the sample, i.e.:

$P(S^C \cap L^C) = \frac{126}{212} = 0.594.$

Solution to question b.

In the second part, the student should identify the conditional probability $P(S \mid L)$, the probability of being a smoker, with the condition that the person has lung problems. To compute this probability the student has to identify the condition and needs to use the first column of Table 1; therefore $P(S \mid L) = \frac{36}{44} = 0{,}81$. As the result should be transformed to percentage, he must multiply the probability by 100 to obtain 81%.

In Table 2, we analyse the mathematical practices that the student has to perform to solve the problem, according to the steps described previously. Consequently, the student needs to remember some concepts (simple and compound experiment; simple and compound events; dependence; simple probability; conditional and joint probability; complementary events; Laplace's rule and percentage).

The student uses different mathematical language (words and symbols, to express the events, the probabilities and the formulas); he needs to perform procedures (arithmetic operations; computation of different probabilities). He also uses some properties (such as the relationship between the different types of frequencies in the 2x2 table and the idea of favourable and possible cases in the experiments). The student should identify the data of the statement; built and read a contingency table and apply Laplace's rule. In addition, the student has to interpret the problem statement, and transform a probability into a percentage. The student must perform an analysis-synthesis argument to summarize the solution. As shown in Table 2 the simplicity of the problem is only apparent due to the number of mathematical objects and processes that the student should remember and use.

SUMMARY OF RESULTS

As shown, in the analysis of the solution for this problem (see Table 3) this process is complex, and the student should remember to apply and combine the different types of mathematical objects considered in our theoretical framework.

Some of these objects are complex, as shown in previous research. For example, Kelly and Zwiers (1986) described the confusion between independence and mutual exclusiveness: while two mutually exclusive events are essentially dependent, i.e. one cannot occur without the other, the contrary is not generally true.

The language used is always numerical, verbal and symbolic; sometimes graphs, tables or diagrams built by the student are added. As regards the arguments, every problem always requires to carry out an analysis (dividing the statement and the solution in parts) and a synthesis (composing the solution).

The same method of analysis was applied to the 36 problems proposed in the tests in the years 2003, 2008 and 2013. In Table 3 we show the mathematical objects

involved in solving the problems proposed in 2003, which include difficult concepts. For example, some students do not differentiate conditional and joint probabilities (Pollatsek, Well, Konold, & Hardiman, 1987). Other students make frequent errors when applying Bayes' rule (Gras & Totohasina, 1995). These errors may appear when solving some of the problems proposed in the university entrance tests.

<table>
<tr><th></th><th>Item solution</th><th>Analysis</th></tr>
<tr><td>Statement</td><td>Statement: From a total of 212 elderly people in a geriatric residence, 44 of them have lung problems; 78 of them are regular smokers, and only 8 have lung disease and are non-smokers</td><td>– Data interpretation (procedure).
– Simple events, simple and composed experiment, sample space for each experiment (concepts).</td></tr>
<tr><td>Problem data</td><td><table><tr><th></th><th>L</th><th>L^C</th><th>TOTAL</th></tr><tr><td>S</td><td>36</td><td>42</td><td>78</td></tr><tr><td>S^C</td><td>8</td><td>126</td><td>134</td></tr><tr><td>TOTAL</td><td>44</td><td>168</td><td>212</td></tr></table></td><td>– Recognizing the problem data; building a contingency table; computing absolute frequencies in some cells (procedures, concepts and representation).</td></tr>
<tr><td>Question a</td><td>$P(S^C \cap L) = \frac{126}{212} = 0.594.$</td><td>– Identifying a joint probability in the statement (interpretation).
– Identifying absolute frequencies (favourable cases) in the contingency table.
– Identifying the total sample (possible cases); procedure and interpretation.
– Applying Laplace's rule (procedure).</td></tr>
<tr><td>Question b</td><td>$P(S|L) = \frac{36}{44} = 0.81; 81\%$</td><td>– Identifying a conditional probability and the condition in the problem statement; identifying the favourable and possible cases in the first column of the table (concepts procedure and interpretation).
– Applying Laplace's rule (procedure).
– Moving from probability to percentage (procedure).</td></tr>
</table>

Table 2: Analysis of the correct solution of the item P1A.

We consider that the student should enumerate the sample space when it is explicitly requested; the computation of a probability may be simple, compound or conditioned and may refer to a simple or compound event in a simple or compound experiment. Sometimes the problem solution requires the computation of the probability of the complementary of an event, the probability of the union of the compatible events or the formula of conditional probability. Other times it is also necessary to apply total probability or Morgan's rules.

	Problem											
Content	1A	1B	2A	2B	3A	3B	4A	4B	5A	5B	6A	6B
Sample space			x					x	x			
Simple event	x	x	x	x	x	x	x	x	x	x	x	x
Compound event	x			x	x		x	x		x		x
Complementary	x					x	x		x		x	x
Laws of Morgan							x					
Events union						x						
Simple probability	x	x	x	x	x	X	x	x		x	x	x
Laplace rule			x		x			x	x	x		x
Compound experiment	x	x	x	x	x		x	x	x	x	x	x
Joint probability	x	x		x	x	X	x	x	x	x	x	x
Conditioned probability	x	x		x	x	X	x		x	x	x	x
Conditional probability	x	x		x	x	X	x			x	x	x
Dependence	x	x		x	x			x	x		x	x
Independence			x			X	x	x		x	x	
Total probability	x			x	x					x	x	x
Bayes rule	x	x		x	x		x					x

Table 3: Concepts, properties and procedures in the items of 2003.

In order to compare the presence of mathematical objects in the tests in the years 2003, 2008 and 2013 we present Figures 1 and 2. In Figure 1 we analyse half the mathematical objects considered and the rest of them in Figure 2. The purpose of using two figures is to facilitate the analysis of our results.

The enumeration of the sample space is hardly requested explicitly, although in fact the student usually have to form the sample space to continue the solving process.

While in 2003 and 2008 every problem involved computing the probability of a simple event in 2013 no problem proposed this computation. On the contrary the number of problems that asked to compute the probability of compound events was larger in 2013.

Morgan's laws had to be used mostly in the problems proposed in 2008 and also in 8 problems (of 12) in 2013. Laplace's rule should have been directly applied in all the 2008 and 2013 problems and only in half the problems proposed in 2003 (in the remaining problems the probability should be computed using a formula; for example, the product rule).

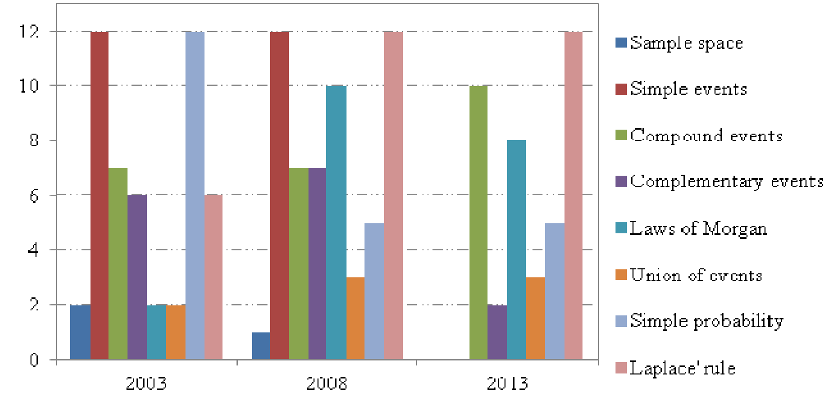

Figure 1: Mathematical objects on problems.

In Figure 2 we highlight the variation of problems related to compound experiments (decreasing with time), although there is no much change in the presence of joint and conditional probability, because sometimes these problems are set in simple experiments.

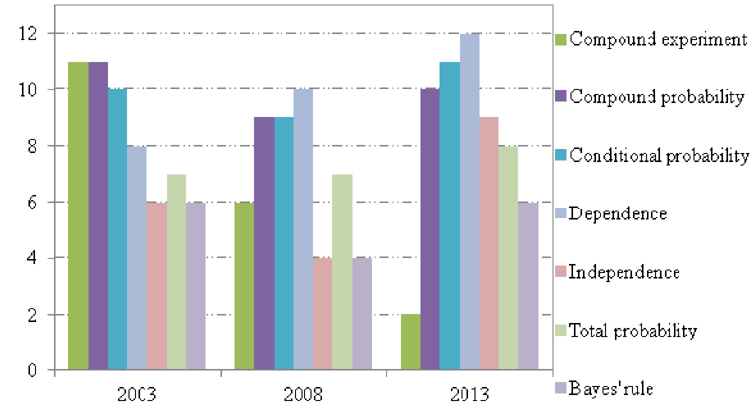

Figure 2: Mathematical objects on problems.

Contrary to Ortiz (1999) research, we observe a strong presence of conditional probability problems, which involve numerous reasoning biases (Diaz, Contreras, Batanero, & Roa, 2012). Therefore this presence implies a wish to increase the difficulty of the problems. There is variation in the consideration of independence or dependence in the problems. Finally, since the Bayes theorem always involve the use of total probability, this last topic is widespread in the problems.

CONCLUSIONS

All the university entrance tests analysed include a probability problem (one of the four problems proposed to the students) related to compound or joint probability, and, therefore, these concepts are given great relevance in these tests. The high school curriculum in Spain includes other probability content, such as the central limit theorem; the normal approximation to the binomial distribution; the law of large numbers. Although the tests also include another probability problem related to inference (statistical tests or confidence intervals), it is clear that they give higher relevance joint and conditional probability than to other content.

When analysing the mathematical objects included in the solution, we noticed that the problems proposed in these tests are quite complex; they include many mathematical objects and only in few examples request to compute a simple probability. Furthermore, in these particular cases the students are asked to provide proof for some abstract properties of probability.

An indicator of the difficulty of the problems is the large proportion of them where no application context is included; that is, problems that ask the student to calculate, simplify expressions or provide justification of abstract probabilities. Probability knowledge and reasoning is needed in everyday or professional situations and the assessment instruments reflect the type of content we want the students to learn. In problems with no context, the multiple applications of probability to different sciences are hidden to the students and their probability reasoning is reduced to algebraic reasoning. Moreover, most problems require to work with dependent experiments; apply a decomposition of conditional or compound probabilities; use the total probability or the Bayes' rules or both.

To conclude, our analysis reveals the high difficulty of the probability problems proposed to the students in the university entrance tests. The scores obtained by the students in these tests often determine that they can enter their preferred career; consequently, our results should be taken into account by the test designers in order to build more reasonable assessment tests in the future.

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SEMIOTIC COMPLEXITY LEVELS AND ACTIVITIES RELATED TO STATISTICAL GRAPHS IN CHILEAN PRIMARY EDUCATION TEXTBOOKS

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In this work we analyze the activities proposed and the semiotic complexity levels in the statistical graphs included in two series of Chilean primary education textbooks using content analysis. The most common activities related to these graphs in these textbooks are performing computations, building graphs and using the graphs as examples. The most common complexity level was level 3 (representing a data distribution). These results also show (with smaller frequency) a varied type of activity and higher semiotic levels with increasing age of students.

INTRODUCTION

The scientific and technological development of modern society requires an educational process that helps to promote the understanding of information, which is becoming more abundant and specialized every day.

Statistical graphs play an important role in the communication and dissemination of this information (Monroy, 2007). According to Arteaga, Batanero, Cañadas, and Contreras (2011) these graphs are widely used in different situations of daily life. Carvalho (2011) remarks that the presence of graphs, tables and statistical summaries in the media is aimed at summarizing information and facilitating its understanding; however, these representations are often used to get some benefit out of particular situations, and include biased information. Consequently it is important that the citizen knows how to read and interpret graphs, to ensure his/her adequate integration into our society (Arteaga, Batanero, & Contreras, 2011).

This need is reflected in the relevance given to statistical literacy, which Gal (2002) defined as the union of two competences:

> a) People's ability to interpret and critically evaluate statistical information, data-related arguments, or stochastic phenomena, which they may encounter in diverse contexts; … and, when relevant; (b) their ability to discuss or communicate their reactions to such statistical information, such as their understanding of the meaning of the information, their opinions about the implications of this information (p. 2-3).

Del Pino and Estrella (2012) compare statistical literacy to a civil right, since people should be able to use elementary arguments, language and tools to act in an informed and critical way in the knowledgeable society.

These demands for statistical culture have been considered in various curricular documents, e.g., The GAISE project (Franklin, Kader, Mewborn, Moreno, Peck, Perry, & Scheaffer, 2007), the Common Core State Standards Initiative (CCSSI,

2010), the Spanish regulations of the Ministry of Education, Culture and Sports (MECD, 2014) and the Chilean regulations of the Ministry of Education (MINEDUC, 2012). These documents emphasize the relevance of statistical graphs, which represent data coming from familiar situations and contexts where children participate actively. Thus, as shown in the Chilean curricular guidelines (MINEDUC, 2012), statistical graphs are approached throughout primary education from the first level with increasing sophistication and with constant work (Díaz-Levicoy, 2014).

This background led us to start a research project intended to analyze the way in which the statistical graphs are introduced in the Chilean primary education textbooks, identify the main variables that may affect their difficulty and use this information to help teachers to organize the teaching of the topic. Below we present the study background and some preliminary results.

BACKGROUND

Statistical graphs

Statistical graphs have been analyzed from various points of view. Bertin (1967) remarked that a statistical graph is a complex semiotic system, since its understanding requires the local interpretation of each element, as well as a global interpretation of the graph. Moreover, Arteaga (2011), and Batanero, Arteaga, and Ruiz (2010) analyzed the semiotic activity involved in the construction of graphs, that varies in complexity, according to the mathematical objects needed in this process. Taking into account this activity, the authors described four semiotic levels in statistical graphs that will be used in our analysis (with examples reproduced in Figure 1):

- *Representing isolated data.* At this level, the graph only represents some isolated data (either individual data or a few data), without considering the whole set from which the data were extracted. In these graphs, the author does not need to consider the ideas of variable or distribution, since no comprehensive analysis of a whole data set is performed. In the example reproduced in Figure 1, only a dot is represented in a scatter plot.
- *Representing a data set without building a distribution.* At this level a list of data or a data set is represented in the same order in which the data were collected. There is no grouping of similar values of the variable or computation of frequencies. Consequently, although in this graph the idea of variable is used, the distribution is absent. Moreover, often the order of the data in the graph is not a numerical order, but is artificial. In Figure 1 the speed in each kilometre is represented with no order (of these speeds).
- *Representing a data distribution.* These graphs include the representation of a distribution, with values and frequencies for each value; the order of the variable values in the graph axes (if used) is the ordinary numerical order. The example in Figure 1 shows the frequencies of people going to the movies in the morning,

afternoon and night. So there is a distribution for a qualitative variable (time to go to the movies).

- *Representing several distributions on the same graph.* At the highest level more than one distribution is represented on the same graph. This often requires a decision on a common scale that makes the graph understandable. In the example in Figure 1, the distribution of the type of food taken by children in two different classes (two distributions) is compared on the same graph.

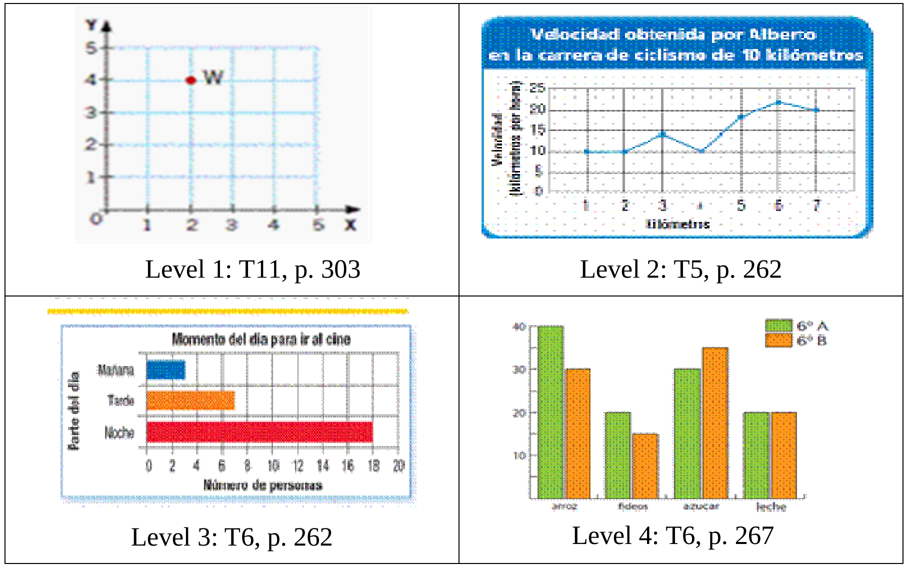

Figure 1. Different levels of semiotic complexity in the graphs included in the textbooks (Ti, 1≤i≤12).

Textbooks

There is increasing research on textbooks in different areas of learning and mathematics is not an exception. Despite this research tradition, the analyses of statistics and probability textbooks are scarce, and the analysis of statistical graphs in the textbooks even more so.

A first study related to this topic is that by May (2009), who studied the statistical graphs in 25 university textbooks directed towards psychologists. The most common graphs in these books were the line graph, histogram, density curve and bar graph; most activities were directed towards the construction of graphs and the reading level in Curcio's classification (Curcio, 1989) was reading the data. According to these authors these activities promote statistical literacy but not statistical reasoning.

Gómez, Ortiz, Batanero, and Contreras (2013) studied the language of probability in two series of Spanish primary education textbooks. Their results suggest an extensive

use of tabular and graphical representations (bar graph, sectors, pictogram and histogram); the authors did not analyse the variables that characterize these graphs.

Gea, Batanero, Arteaga, Cañadas, and Contreras (2014) analysed the presentation of correlation and regression in 8 Spanish high school textbooks. They concluded that the main graph used is the scatter plots around which many exercises are set and in which the students should analyze the sign and strength of linear relationships. The authors also found some bar charts, three-dimensional histograms and bubble graphs in these textbooks.

Mateus (2014) analyzed 5 Colombian textbooks (primary and secondary school levels). These textbooks showed a predominance of contexts related to the students' life; the activities included some basic construction, and elements of reading graphs; the authors also identified some errors in the graphs.

Díaz-Levicoy (2014) studied the statistical graphs on a sample of 18 Spanish Primary Education textbooks. He found that the most common representation was the bar graph; usually the reading level in Curcio (1989) classification was "reading into the data"; the semiotic level in the graph was "representation of a data distribution" and the most common activity was building a graph or performing computations from data represented in the graph. Our research is intended to complement these studies; in particular to compare the analysis of Chilean textbooks with our previous study of Spanish textbooks for the same educational level (Díaz-Levicoy, 2014).

METHOD

In this paper we follow a qualitative methodology, based on content analysis (López, 2002). The sample consists of 12 primary education textbooks that were selected by intentional sampling, based on a controlled selection and according to certain characteristics that have been previously defined. More precisely, we selected textbooks that were widely used in the schools in the academic year 2013-2014 and edited according to the curriculum guidelines for these courses. The list of the textbooks used in this research is included as an appendix.

We considered all the textbook sections that included any statistical graphs (exercises, examples, definitions, problems) or that involved any activity related to statistical graphs. Each paragraph where one or more statistical graphs intervene was an analysis unit. A total of 421 different paragraphs were identified and analysed; for each of them the categories in the relevant variables were coded; in this case we considered the variables "semiotic complexity levels" and "type of activity". All the data were recorded in SPSS and the data file was analysed to obtain some conclusions.

RESULTS

Semiotic complexity levels

In Table 1 we classify the activities analyzed according to the semiotic level of the graph involved in the same (taking the higher level, in case the activity involves two distributions). The predominance of level 3 is clear (representing a distribution in the graph). Consequently in the majority of activities the children have to work with a data distribution (a 66% of the activities) that is, with graphs representing one or more distributions (if we take into account levels 3 and 4). There are also 26.1% of activities in level 2, where the data set is represented with no computation of frequencies or building of the distribution.

Level	School level (grade)						
	1 (n=43)	2 (n=65)	3 (n=95)	4 (n=38)	5 (n=87)	6 (n=93)	Total (n=421)
1					2.3	1.1	0.7
2	44.2	33.8	30.5	15.8	26.4	11.8	26.1
3	55.8	66.2	69.5	84.2	58.6	66.7	66
4					12.6	20.4	7.1

Table 1: Percentage of activities in the texts analyzed according to their semiotic complexity level.

When we analyse this variable by school year we observe that in the first four grades we only found levels 2 and 3 statistical graphs, with constant predominance of level 3, which gradually increase in these four levels. In our opinion it would be reasonable to include more level 2 graphs and even some level 1 graphs for such young children. In grade 5 and 6, we found level 4 graphs, as well as sporadic level 1 activities (just 3 activities, which correspond to scatter plots). These results agree with those of Díaz-Levicoy (2014) in his analysis of Spanish textbooks; although in the Spanish textbooks he found statistical graphs of level 4 from the second level of primary education.

Type of activities

A second variable analysed was the type of activity that children are requested in relation with the statistical graphs. These activities were classified in the following categories that were defined taking into account previous researches:

- *Reading the graph*. In this activity a graph is given and children are asked to read the information displayed in the graph. The activity may involve different reading levels according to Curcio (1989): reading the data; reading between the data; reading beyond the data or reading behind the data. We did not differentiate this variable in this paper.
- *Calculation*. In this case children are given a graph and are asked to perform some comparisons (for example, finding the mode) and / or simple calculations (for example, transforming a frequency to percentage), using the information in a statistical graph; this activity also involves reading the graph and some additional computations.

- *Building a graph*. In this activity children are asked to build a statistical graph using data presented in statistical tables or lists.
- *Complete a graph*. Similar to the previous activity; but in this case the children finish the construction of a statistical graph according to a pattern given (the first steps are presented to the children).
- *Translating*. In this activity the children should build a new a graph with the information provided in another, or translate the graph to a table.
- *Example*. This is the section of the textbook that uses a graph to clarify ideas and / or concepts.
- *Invent a problem*. In this type of activity the children should create a problem that makes sense for data presented in a statistical graph, i.e., they should generate a context where the data are relevant and coherent.
- *Comparing and justifying*. Students have to select a graph according to the nature of the data, pointing out the advantages and disadvantages of the specific graph selected, and indicate the best form of presenting the information (in a table or graph).

More than one type of activity can be asked from a statistical graph (e.g., some activity ask to build a graph and later calculate from the graph); in these cases to perform this analysis we considered these activities separately.

In Table 2 we show the distribution of the types of activities that we found after the analysis of these textbooks. We observe that the most frequent activities are computing (54.2%), building (19.2%), examples (15.2%), comparing and justifying (9.7%) and reading (7.6%).

Reading, building, examples and comparing and justifying are included in all the school levels. The activity "completing graphs" is presented in the first levels to guide the students when they are learning to build statistical graphs. The percentage of computation activities is high and increases with school level. We remark that this type of activity does not help students to develop their statistical reasoning or literacy; it is only a mathematical activity where the statistical graphs are only a tool to present data. Our suggestion is that this type of activity should be less frequent and changed by more interpretative activities.

Our results in this variable agree with those of Díaz-Levicoy (2014), where the activities of reading, building and example predominate. An important difference is that computation activities were only 8.8% in the Spanish textbooks while reading activities amounted to 40% and therefore there were more interpretative activities in these textbooks. Another difference is that the examples are presented for the first grade in Chilean texts, while in Spanish it is done from the second grade; finally, in the Spanish books some activities asked to describe the variables included in the graph; while we do not identify these activities in the Chilean text books.

Type of activity	School level (grade) 1 (n=43)	2 (n=65)	3 (n=95)	4 (n=38)	5 (n=87)	6 (n=93)	Total (n=421)
Reading	9.3	12.3	9.5	2.6	6.9	4.3	7.6
Calculation	39.5	47.7	42.1	63.2	59.8	68.8	54.2
Building	20.9	20	30.5	10.5	10.3	18.3	19.2
Completing	9.3	6.2	2.1				2.4
Translating	9.3		1.1			1.1	1.4
Example	14	18.5	15.8	21.1	13.8	11.8	15.2
Inv. Problems			2.1	2.6	1.1		1
Comparing and justifying	9.3	1.5	7.4	23.7	11.5	10.8	9.7

Table 2: Percentages of tasks in primary education textbooks according to the requested activity.

CONCLUSIONS

Textbooks are a teaching and learning resource with an important tradition within the classroom, because they provide support to teachers and students throughout the instructional process. It is therefore necessary to study how the textbooks present mathematical and statistical topics as a first step to suggest possible improvement in their content and to check that they follow the curricular guidelines.

Statistical graphs are a new topic in Chilean primary education and for this reason our analysis is particularly needed. A first conclusion is the large number of activities (421 as compared with 215 activities in the same number of Spanish textbooks for these curricular levels). Although this number (double proportion of graph activities in the Chilean books as compared with the Spanish books) provides the teacher with rich material, it is also important to guide the teacher with criteria to select those that will be used in the classroom, since it is clear that it is impossible for the children to complete such a large number of tasks.

The two common semiotic complexity levels in these books are "representation of a data distribution" and "representation of a dataset without producing the distribution (90% of all the activities). Although these two levels are appropriate for the upper levels of primary education, and even level 4 (representing two or more distributions in the same graph), we recommend to include some more simple level 1 situations for the grades 1 and 2 as they appear in the Spanish texts.

We also recommend diminishing the emphasis on computation and reinforcing other activities relevant for learning. For example, the activity of "completing a graph" should be given more emphasis at all primary school levels, because this activity helps students to become familiar with new graphs as they are introduced in the curriculum. The “reading” activity should have a higher presence in the early levels, starting with simple “reading the data”, and progressing through the levels of “reading between the data”, “reading beyond the data” and “reading behind the data” in Curcio (1989) framework throughout these school years. In the same way, the activities of "inventing problems" and "translating graphs" should have greater presence as they are more cognitively challenging for students and reinforce their statistical reasoning.

We hope our results are useful for teachers and teacher educators, who have the responsibility to make statistical literacy a reality for everyone and to develop statistical sense (Batanero, Díaz, Contreras, & Roa, 2013) in children and teachers.

ACKNOWLEDGEMENT

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APPENDIX: TEXBOOKS ANALYSED

T1. Salazar, R., & Sprovera, M. (2014). *Matemática 1º Básico. Texto del Estudiante*. Santiago: Fe y Alegría.

T2. Ho Kheong, F., Ramakrishnan, C., Pui Wah, B. L., & Choo, M. (2014). *Mi Matemática. Texto del Estudiante 2°*. Santiago: Marshall Cavendish.

T3. Charles, R., Caldwell, J., Cavanagh, M., Chancellor, D., Copley, J., Crown, W., Fennell, F., Ramirez, A., Sammons, K., Schielack, J., Tate, W., & Van de Walle, J. (2014). *Matemática 3º Educación Básica. Texto del estudiante*. Santiago: Pearson.

T4. Andrews, A., Dixon, J., Norwood, K., Roby, T., Scheer, J., Bennett, J., Luckie, L., Newman, V., Scarcella, R., & Wright, D. (2014a). *Matemática 4º Básico. Texto del estudiante*. Santiago: Galileo.

T5. Andrews, A., Dixon, J., Norwood, K., Roby, T., Scheer, J., Bennett, J., Luckie, L., Newman, V., Scarcella, R., & Wright, D. (2014b). *Matemática 5º Básico. Texto del estudiante*. Santiago: Galileo.

T6. Andrews, A., Dixon, J., Norwood, K., Roby, T., Scheer, J., Bennett, J., Luckie, L., Newman, V., Scarcella, R., & Wright, D. (2014c). *Matemática 6º Básico. Texto del estudiante*. Santiago: Galileo.

T7. Baeza, A., López, F., Sandoval, M., & Urra, A. (2013). *Matemática 1º Básico. Tomo II*. Santiago: Santillana.

T8. Baeza, A., Blajtrach, P., Kükenshöner, C., & Sandoval, M. (2013). *Matemática 2º Básico. Tomo II*. Santiago: Santillana.

T9. Véliz, C. (2013). *Matemática 3º Básico. Tomo II*. Santiago: Santillana.

T10. Batarce, Y., Cáceres, B., & Kükenshöner, C. (2013). *Matemática 4º Básico. Tomo II*. Santiago: Santillana.

T11. Ávila, J., Fuenzalida, C., Jiménez, M., & Ramírez, P. (2013). *Matemática 5º Básico. Tomo II*. Santiago: Santillana.

T12. Ávila, J., Castro, C., Merino, R., & Ramírez, P. (2013). *Matemática 6º Básico. Tomo II*. Santiago: Santillana.

USING MODELS AND MODELING TO SUPPORT THE DEVELOPMENT OF COLLEGE-LEVEL STUDENTS' REASONING ABOUT STATISTICAL INFERENCE

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The article describes a teaching experiment that took place in a post-graduate quantitative research methods course, which puts models and modelling at the core of the curriculum. Using the affordances of the dynamic software TinkerPlots2, students engaged in various open-ended Model-Eliciting Activities (MEAs) in which they had to create and test statistical models and use them to solve complex, real world problems. Findings indicate that the focus of instruction on modelling and simulation, using TinkerPlots2 as an investigation tool, fostered students' ability to reason about the stochastic. Through building models and using them to evaluate research claims and hypotheses, graduate students in our study developed relatively coherent understandings of fundamental concepts related to statistical inference.

INTRODUCTION

Although statistical inference is the cornerstone of modern statistical methods, grasping the key ideas related to inferential statistics is a known area of difficulties for learners, at all educational levels (e.g., Garfield & Ahlgren, 1998; Rubin, Hammerman, & Konold, 2006). Research has long suggested that students have difficulty using inferential statistics methods appropriately in applied problems. For example, research on introductory college-level statistics courses suggests that even students who can successfully implement procedures for hypothesis testing and parameter estimation are often unable to use these procedures appropriately in applications (e.g., delMas, Garfield, Ooms, & Chance, 2007; Gardner & Hudson, 1999). As Erickson (2006) points out, "inference is so hard that even professional researchers use it inappropriately" (p. 1).

Traditionally, introductory statistics courses adopt a linear, hierarchical approach to the different statistical topics encountered in the course. The structure of almost every introductory statistics course is to first start with descriptive and exploratory data analysis, then move into probability, and finally go to inferential statistics. Statistical inference is presented as a set of formal tests and procedures, through which information contained in sample data is used to either estimate the respective population parameters (i.e., construct confidence intervals), or to check claims made regarding the values of population parameters (i.e., perform hypothesis testing).

The article shares some of the experiences from a teaching experiment in a graduate level, quantitative research methods course, which adopted an informal, data-driven approach to statistics instruction, with models and modelling at the core of the curriculum. Using the dynamic statistics software TinkerPlots2 as an investigation tool, the study sought to answer the following question:

How can the model building affordances provided by a technological learning environment like TinkerPlots2, be utilized in instruction to scaffold and extend students' reasoning about key ideas related to statistical inference?

LITERATURE REVIEW

Models and modelling are paramount to understanding the core logic of statistical inference. The appearance in recent years of dynamic statistics learning environments with model eliciting affordances (e.g., TinkerPlots2 (Konold & Miller, 2011), and Fathom© (Finzer, 2002)), have provided an enormous potential for making inferential reasoning accessible to students. These software are designed explicitly to support integration of exploratory data analysis approaches and probabilistic models, and to allow for model generation and experimentation (e.g., drawing samples from a model, improving models, conducting simulations). They provide a medium for the design of activities that integrate experiential and formal pieces of knowledge, allowing students to make direct connections between physical experience and its formal representations (Paparistodemou, Noss, & Pratt, 2008). Students can build models to evaluate claims and hypothesis. They can articulate their informal theories and use them to make conjectures, and then use experimental results to test and modify these conjectures.

Several researchers have, in recent years, been exploiting the model-eliciting affordances provided by dynamic statistics software for promoting learners' ability to reason and argue about data-based inferences, with very encouraging results (e.g., Biehler & Prömmel, 2010; Erickson, 2013; Garfield, delMas, & Zieffler, 2012; Konold & Lehrer, 2008; Meletiou-Mavrotheris, 2003). Some of the conducted studies have demonstrated that even young children can develop powerful notions about inference when using appropriate data visualization tools (e.g., Meletiou-Mavrotheris & Paparistodemou, 2015).

METHODOLOGY

Context and Participants

The teaching experiment took place in a Quantitative Research Methods course targeting graduate students enrolled in an M.A. in Educational Studies program. Class met once a week, for three hours each time, over a period of four months. The first author was the course instructor. There were nineteen ($n=19$) students enrolled in the course. Participants were either pre-service or in-service teachers, who were characterized by diversity in a number of parameters including age, and professional and academic background. Their age ranged from 23 to 42. Some had an academic background in primary education, while the rest were secondary school teachers in different domains (languages, humanities, natural sciences, physical sciences etc.). Students had a varied background in statistics. Most of the older participants had very limited exposure to statistics prior to the course and had never formally studied the subject, while the younger ones had typically completed an introductory statistics course while at college. However, even those students who had formally studied statistics in the past, had attended traditional lecture-based statistics courses that made minimal use of technology. Thus, upon entering the course, almost all of the students had very weak statistical reasoning and a tendency to focus on the procedural aspects of statistics.

Course Curriculum and Didactical Approach

The study employed a non-conventional approach to the teaching of the Quantitative Research Methods, which put models and modelling at the core of the curriculum. In designing the teaching experiment, we ensured that our intervention covered the set curriculum included in the course syllabus. However, we expanded the curriculum by including, throughout the semester, activities that aimed at raising students' awareness of models and modelling, and of their usefulness in research settings involving statistical investigations with stochastic phenomena.

The course focus on modelling—along with inference—was being facilitated by having students use the dynamical statistical software package TinkerPlots2 for all modelling and analysis. The class meetings took place in the computer lab. Throughout the semester, students were using TinkerPlots2 to work on a set of carefully designed open-ended Model-Eliciting Activities (MEAs) (Lesh et al., 2000) in which they had to create and test statistical models in order to solve complex, real world problem of statistics and provide answers to their research questions (Garfield, delMas, & Zieffler, 2010). The activities were carefully designed to support but, at the same time, also explore students' evolving understandings of the fundamental ideas related to statistical inference in the context of engaging in models and modelling for simulating data and evaluating their research claims and hypotheses. Some of the MEAs were completed individually, and others collaboratively in groups of 3-4 students. The MEA *"Helper or Hinderer"* (adapted from http://www.tc.umn.edu/~catalst/materials.php), shown in Figure 1, is a typical example of the activities in which students engaged during the course.

Methods of Data Collection and Analysis

In examining students' learning progress, a variety of both qualitative and quantitative data gathering techniques were used: classroom observation, videotaping, interviews of selected students, and student work samples.

Through careful analysis of the collected data, we documented the ways in which ideas related to models and modelling were understood and utilized by students in the context of making informal statistical inferences from data, using Tinkerplots2 as an investigation tool. We examined how learners' endeavours with TinkerPlots2, but also with MEAs, impacted the development of their understanding of statistical inference from the simplest forms (informal) to the more complex ones (formal).

You all recognize the difference between naughty and nice, right? What about children less than a year old–do they recognize the difference and show a preference for nice over naughty? In a study reported in the November 2007 issue of Nature, researchers investigated whether infants take into account an individual's actions towards others in evaluating that individual as appealing or aversive, perhaps laying for the foundation for social interaction. In one component of the study, 10-month-old infants were shown a "climber" character (a piece of wood with "google" eyes glued onto it) that could not make it up a hill in two tries. Then they were alternately shown two scenarios for the climber's next try, one where the climber was pushed to the top of the hill by another character ("helper") and one where the climber was pushed back down the hill by another character ("hinderer"). The infant was alternately shown these two scenarios several times. Then the child was presented with both pieces of wood (the helper and the hinderer) and asked to pick one to play with. The researchers found that the 14 of the 16 infants chose the helper over the hinderer.

Research Question: **Are infants able to notice and react to helpful or hindering behavior observed in others?**

1. What proportion of these infants chose the helper toy?
2. What does that suggest about the answer to the research question? Explain.
3. Suppose for the moment that the researchers' conjecture is wrong, and infants do not really show any preference for either type of toy. In other words, infants just blindly pick one toy or the other, without any regard for whether it was the helper toy or the hinderer. This is the model based on random chance. If this is really the case (that infants show no preference between the helper and hinderer), is it possible that 14 out of 16 infants could have chosen the helper toy just by chance?
4. Would the observed result (14 of 16 choosing the helper) be surprising if infants had no real preference or not? How strong do you believe the evidence is against the chance model?
5. You are going to set up a Tinkerplots simulation to model the process of 16 hypothetical infants making their selections using random chance. The model in this situation–and in many other situations–reflects the assumption of no preference, or nothing affecting their selection other than random chance. Because of this, statisticians sometimes refer to the model as the **null model**. Write a brief description of the null model you will build based on the assumption of no preference. Be sure that you explicitly identify each of the following in your description: number of trials, potential outcomes of a single trial; probabilities of the potential outcomes; sampling with or without replacement.
6. Based on your description of the model, set up the model in TinkerPlots
7. Carry out the simulation study to obtain a large number simulated outcomes when the infants really have no preference and are simply picking a toy at random. Keep track of the number that pick the Helper toy in each experiment, and sketch in the results of your simulation study below.
8. Is the observed result from the original experiment likely or unlikely under the null model? What does this suggest about the null model? Explain.
9. Quantify the strength of evidence for the observed result (14 out of 16 infants choosing helper toy).
10. Based on your analysis, how strong is the evidence against the null model?
11. What does this suggest about infants making their selections based only on random chance?
12. Is there a theoretical distribution, which could be used to model the experiment? Justify your answer.
13. What assumptions do you need to make? Are these assumptions the same or different to the assumptions you made for the simulation? How likely are they to be true?
14. What are the parameter(s) of the distribution?
15. Using this distribution, estimate the probability of at least 14 out of 16 infants randomly choosing the helper toy.
16. Compare these probabilities with your simulation.
17. To what extent has Tinkerplots helped you (or not) to construct the theoretical probabilistic distribution?

Figure 1: "Helper or Hinderer" Model-Eliciting Activity.

RESULTS

The teaching experiment focused on building conceptual understanding of the big picture of statistics rather than presenting statistical content as a sequenced list of curricular topics. In order to provide an authentic model of the statistical culture, instruction encouraged statistical inquiry and data modelling rather than teaching methods and procedures in isolation (Lehrer & Schauble, 2005).

A progressive formalization approach was employed. The first part of the course focused on building a teaching pathway towards formal inference by helping students experience and develop the 'big ideas' of informal inference. Through their engagement with the open-ended MEAs, students learned where these ideas apply and how. Later in the course, students were introduced to confirmatory or formal inference methods, and began comparing empirical probabilities with the theoretical ones. They learned the formal procedures for building sampling distributions, constructing confidence intervals, and conducting hypothesis testing.

The course content and structure was such that it encouraged "statistical enculturation". Statistical thinking was presented as a synthesis of statistical knowledge, context knowledge, and the information in the data in order to produce implications and insights, and to test and refine conjectures. The MEAs put students in a variety of authentic contexts where they needed statistical tools to model and make sense of the situation at hand. Probability was not presented as a body of clear and unambiguous generalizations free of any concrete interpretations, but as a modelling tool. Probability distributions were presented as models based on some assumptions which, when changed, might lead to changes in the distribution. The emphasis was not on teaching their formal properties, but on helping students understand why and where one could use these probability distributions to model a certain phenomenon, and in what ways this is useful. The similarities and differences between these ideal, mathematical models of reality, and statistical models based on experimental data were emphasized throughout the course. From informal uses of models early in the course to formal uses as part of significance tests at the later part the course, we were encouraging explicit discussion of how every model is essentially an oversimplification of reality which involves loss of information, and of how the success of probability models depends on their practically, and on their potential to give useful answers to our research questions.

The "Helper or Hinderer" MEA shown in Figure 1 is a characteristic example of the type of activities in which students engaged during the earlier parts of the teaching experiment. This activity, which is briefly described next, served as an informal introduction to the logic and process of hypothesis testing.

"Helper or Hinderer" MEA

The "Helper or Hinderer" MEA is based on an actual research study reported in a November 2007 issue of *Nature* (Hamlin, Wynn, & Bloom, 2007), in which researchers investigated whether infants take into account an individual's actions

towards others in evaluating that individual as appealing or aversive. Sixteen (n=16) 10-month-old infants were shown a "climber" character that could not make it up a hill in two tries. Then they were alternately shown two scenarios for the climber's next try: a scenario where the climber was pushed to the top of the hill by another character ("helper") and a scenario where the climber was pushed back down the hill by another character ("hinderer"). This was repeated several times. Then the infant was presented with both characters and was asked to pick one to play with. The researchers found that the 14 of the 16 infants chose the helper over the hinderer.

After reading and discussing the scenario with the other members in their group, students concluded that the experimental result provided convincing evidence that infants had genuine preference for the "helper" toy. They all considered the fact that 87.5 percent of the infants (14 out of 16) chose the "helper" character as strong indication that children are able to notice and react to helpful or hindering behaviour: "*P = 0.875 is strong indication that the findings have not occurred by chance, i.e. that children did not just randomly pick among the two types of characters*". They all thought that the observed result would be very unlikely to occur if infants had just blindly picked a toy: "*There is strong evidence against the null model of no real preference between helper and hinderer*"; "*Babies seem able to evaluate others' behaviour and to show preference towards characters with helpful behaviour*".

Next, each group of students moved to TinkerPlots2 in order to model the experiment and find the likelihood of the observed result (14 or more infants choosing the helper character) to have occurred by chance. The need to properly model the situation forced students to think carefully about the problem and its context. All groups initially faced some conceptual difficulties in modelling the problem at hand, eventually however they were all able to appropriate model the situation based on the null model of no real preference among infants. They first simulated a single sample of 16 hypothetical infants under the null model and repeated the process of simulating a single trial several times (see Figure 2):

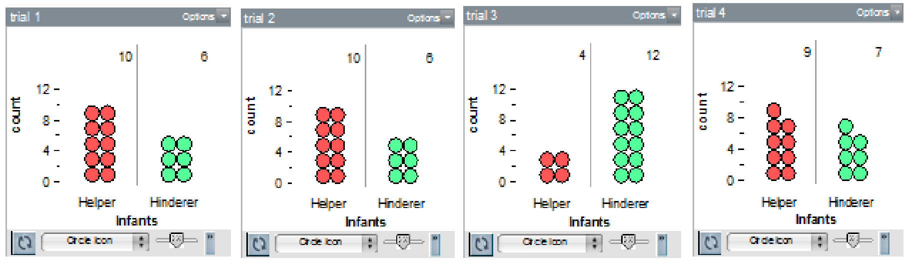

Figure 2: Outcomes of single sample simulations, under the null model, of the "Helper or Hinderer" MEA.

Comparing of the results obtained from different repetitions of the experiment, helped students to begin developing some sense of the expected variability in the number of infants picking the "helper" toy: "*Results vary but 14 or more infants out* *of 16 selecting the "helper" character never occurs. Thus, it seems that infants really have a preference and are not just picking a toy at random"*.

Acknowledging the need to repeat the experiment a very large number of trials to draw safer conclusions, students next chose the "Collect Statistic" feature of TinkerPlots2 to keep track of the number of students picking the "Helper" toy each time. They next asked the software to repeat the experiment a large number of times and to draw the resulting distribution of sample statistics. In Figure 3, for example, we see the distribution of sample statistics drawn by a group of students who repeated the experiment 2000 times.

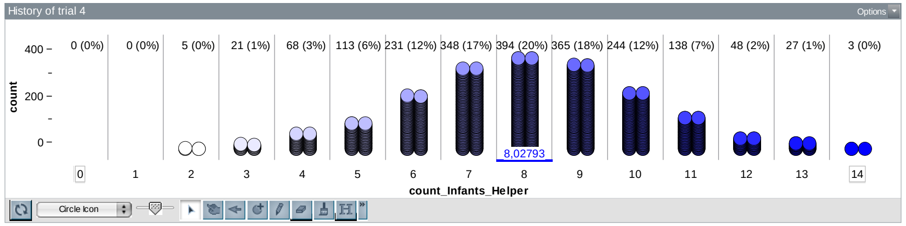

Figure 3: Histogram of sample statistics for the "Helper or Hinderer" MEA - A measures collection of 2000 samples.

Looking at the histogram of sample statistics, students made observations such as, for example, that *"a number as high as 14 or higher is very rare since almost all numbers in the graph are smaller than 14"*. They also noted that *"the distribution of the collected statistics is normal and its center is close to 8 which is the mean expected value if we assume no real preference among infants."* Thus, students' engagement with this activity enabled them to grapple with and to intuitively grasp challenging inferential concepts such as the p-value and the central limit theorem.

Finally, students used the properties of the binomial distribution to determine the theoretical probability of at least 14 out of 16 infants randomly choosing the "helper" toy under the chance model. They found this probability to be very small (p=0.0021), and very close to the empirical results they had obtained using the computer simulation. They again concluded that it was not merely a coincidence that so many infants picked the helper toy:

> "Based on our analysis, there is strong evidence against the null model. Looking at both the empirical results and the theoretical probabilities, it becomes obvious that infants are able to recognize helpful or obstructive behavior towards others and to show preference for the helpful toy"; "The experiment showed that infants are not making their selection based on chance, but tend to choose the helper toy. The reasons for this tendency warrant further investigation by researchers."

The study participants engaged in several MEAs similar in nature and format to the "Helper or Hinderer" task. Using TinkerPlots2, students built and modified their own models of real world phenomena, and used them to informally test hypotheses and

draw inferences. Their engagement with data-driven inferences helped them to develop sound informal understanding of the logic of hypothesis testing and its related statistical ideas (significance level, p-value, null and alternative hypothesis etc.), and served as a foundation for the formal study of inferential statistics.

Despite the fact that the simulation approach did give students rich insights that helped them better understand and appreciate the meaning and power of fundamental inferential concepts such as the central limit theorem, sampling distribution, and hypothesis testing, the introduction to the formal procedures for conducting hypothesis testing still caused difficulties to several of the study participants. Whereas almost all students in the course seemed very comfortable with the process of informally building distributions of sample statistics (i.e. sampling distributions) and making inferences, several of them got very puzzled and intimidated when abstract notation and procedures were first introduced. In the beginning, students had difficulties in using traditional tools such as z, and t. Eventually, the modelling, simulation-based approach gave them insights that helped them better understand and appreciate the meaning and power of these formal inferential tools. Comparing empirical probabilities with the theoretical ones helped them make direct connections between the formal and the informal. There were some students in the class who, due a poor mathematical background, even at the end of the semester still confused basic statistical notation. However, the conceptual understanding of the logic of statistical inference that even these students had was much better compared to what is typically observed in students completing similar courses that employ more conventional, lecture-based approaches.

CONCLUSIONS

The current study has provided some useful insights as to how use of technology can help to scaffold and extend students' statistical reasoning through the adoption of an informal approach to statistical inference, with models and modelling at its core. The study findings indicate that the didactical approach employed in the teaching experiment, which made modelling, generalization and justification an explicit focus of instruction, fostered participating students' ability to reason and make appropriate data-based inferences. It helped learners develop more coherent mental models of key statistical ideas related to statistical inferences.

Thus, adopting an informal approach to statistical inference, using a technological tool like TinkerPlots2 as a tool for investigating authentic, open-ended model-eliciting activities (MEAs), can help promote powerful ways of thinking statistically, while at the same time also developing students' appreciation for the practical value of statistics (Garfield et al., 2012). Use of the software's model building affordances can stimulate active knowledge construction and extend students' ability to produce reasonable inferences. Of course, the conceptual difficulties students have to overcome when working with experimental probabilities are similar to those they encounter when dealing with theoretical probabilities. They still have to understand the real problem to model it properly. However, the experimental approach gives students a more concrete take on the problems. Through building data models to make sense of the situation at hand, students can actively experiment with statistical ideas, examine the interaction between the data at hand and the theoretical model, and construct more powerful meanings of both informal and formal inferential ideas.

Simulations take longer and empirical probabilities do not exactly match their theoretical counterparts, thus it is still necessary for students to learn streamlined statistics. This is particularly the case for graduate level students like the ones in the current study, who were enrolled in a quantitative research methods course. Nonetheless, because the logic of inference is much more understandable through a data-driven, model-based approach, such an approach should work hand-in-hand with more formal approaches to inferential statistics.

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USING TINKERPLOTS SOFTWARE TO LEARN ABOUT SAMPLING VARIABILITY AND DISTRIBUTIONS AS A BASIS FOR MAKING INFORMAL STATISTICAL INFERENCES

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We report on an instructional study involving the use of TinkerPlots™ software to generate distributions of a sample statistic as a basis for make informal statistical inferences. Activities engaged students in comparing multiple samples of two groups of organisms on a common attribute, with the aim that they: 1) make sense of a TinkerPlots simulation that produced distributions of a sample statistic in context, 2) interpret a sequence of such distributions in relation to increasing sample size, and 3) infer a value of the sampled population attribute. We highlight aspects of students' understandings of a sampling distribution in context, and of their abilities to track the multi-tiered re-sampling process that began with a population and culminated with distributions of the sample statistic on which they based their inferences.

BACKGROUND AND FRAMING

Our information and data-driven age makes statistical decision-making and inference one of the most important schemes of ideas to target in school mathematics instruction (Garfield & Ben-Zvi, 2008). The apparent logic of inference centers on the idea that the value of an attribute of a population of interest — the whole of which is usually not directly accessible — can be inferred only indirectly, by examining the same attribute for a sample that is randomly chosen from that population. In practice, such inferences are typically made on the basis of a single sample drawn from a population. Yet, randomly selected samples have outcomes (values of a particular statistic) that typically vary from sample to sample. As Rubin, Bruce, and Tenney (1991) have argued, the ability to balance and coordinate these two seemingly antithetical ideas — that of individual sample representativeness and the idea of sample-to-sample variability — is key to a coherent understanding of inference. The challenge for instruction, as Rubin and colleagues (1991) frame it, is to have students integrate these contrasting ideas of representativeness and variability into the unified notion of a sampling distribution. Saldanha and Thompson (2007, p. 275) echoed Rubin and colleagues' argument in that "…we do not see how the normative practice of drawing an inference from an individual sample to a population can be understood deeply without reconciling the ideas of sample-to-sample variability and relative frequency patterns that emerge in collections of values of a sample statistic…".

Over the last fifteen years the development of software designed specifically to support the learning of statistics with a focus on interactivity and dynamic visualization (e.g., Finzer, 2012; Konold & Miller, 2011) has made the use of sampling simulations and the generation of associated sampling distributions increasingly common in statistics instruction at the high school level and beyond.

Statistics educators and researchers have explored and recommended the use of simulation-based statistics instruction and curricula to support the development of students' understandings of inference (e.g. Garfield & Ben-Zvi, 2008; Zieffler & Garfield, 2007). This paper reports on an effort in that vein involving a group of high school students' engagement with simulation-based instructional activities using *TinkerPlots*™ (Konold & Miller, 2011) — an interactive and dynamic data exploration and simulation software — to support their ability to make distribution-based informal statistical inferences.

THE STUDY AND INSTRUCTIONAL SEQUENCE

Participants, instructional context, and data corpus

The paper reports on the second phase of an instructional intervention that engaged an intact class of twenty-three 9th-grade students (14 and 15 year-olds) in three 65 to 80-minute lessons in a school located in a suburb of a large city in the Southwestern United States. Instructional activities involved the use of *TinkerPlots* software to explore and analyze data sets in a first phase, and subsequently to simulate re-sampling and use the resulting sampling distributions as a basis for inferring a population parameter's value in a second phase. Students came into the study with some prior statistical knowledge and skills acquired both in their previous coursework and outside of school. For instance, most of the students understood a *sample* to be a "small part" of a larger collection of items that could be used to indicate information about a characteristic of the latter. Students knew how to compute the arithmetic mean of a set of values, and they knew a procedure for finding the median ("the middle value") of a data set. Students also demonstrated an ability to construct and use dot plots and histograms to compare and draw conclusions about two data sets. Students had not previously been exposed to *TinkerPlots*, nor had they previously engaged with re-sampling activities or been exposed to distributions of a sample statistic as products of repeated sampling.

The author oversaw and orchestrated the unfolding of the instructional sequence and class discussions that emerged therein, while an assistant observed and took field notes. Instruction was organized around students' individual work interspersed with small and whole group discussions of ideas addressed in structured activity sheets. The lessons and class discussions were recorded with digital video cameras, and all students' written work on activity sheets and a final exam were recorded. Each student took part in a video-recorded exit interview conducted within one week of the end of the instructional sequence. The overarching research goal was to gain insight into students' thinking and understandings of ideas promoted in instruction in relation to their engagement with that instruction. The data were examined for indications of students' abilities to coordinate and integrate the ideas of sample representativeness and variability into an understanding of sampling distributions, as per Rubin and colleagues (1991) perspective. In particular, the data were examined for evidence of the sense students made of sampling distributions generated by *TinkerPlots* and of whether and how they used them as a basis for making inferences to the sampled population.

The instructional sequence: Lessons 1-3

The phase of the instructional sequence reported here engaged students in an investigation designed to foster their ability to make inferences to a population by considering the variability amongst outcomes of samples of a common size chosen from that population, and by comparing the variability across distributions of a sample statistic generated from collections of such samples of different sizes. The investigation centered on two big statistical ideas: 1) random sampling can be used to draw conclusions about the sampled population, and 2) larger samples lead to sampling distributions that tend to be less variable and hence lead to more confident conclusions about the population. The context for the investigation revolved around the following opening scenario that involved testing whether a species of genetically engineered fish tends to grow longer than normal fish (Key Curriculum Press, 2011):

> *A fish farmer stocked a pond with a new type of genetically engineered fish. The company that supplied the new type claims that these fish will grow to be longer than normal fish. The farmer decided to test the company's claim by stocking the pond with 625 fish, some normal and some genetically engineered. When the fish were fully-grown the farmer caught a sample of 130 fish from the pond and measured the length of each fish in his sample.*

TinkerPlots' sampling simulator tool was introduced and used throughout this part of the sequence to efficiently generate data and collections of random samples of various sizes from the population of fish, and to graphically display the distributions of the resulting sample statistic.

Lesson 1 The first lesson introduced students to the *Fish Farmer* scenario shown above (adapted from Key Curriculum Press, 2011, and Konold, 2005), framing the issue as one of a skeptical fish farmer wanting to test the claim that a genetically engineered type of fish tends to grow longer than a normal type. In the opening part of the investigation, prior to divulging the fish farmer's approach, students were first asked to consider how the farmer might go about testing the claim. The ensuing class discussions focused on issues of data collection and selecting a representative sample, providing occasion for students to consider possible ways in which the farmer might proceed. Students then examined a *TinkerPlots* data file showing the lengths of the two types of fish in the farmer's sample of 130 fish. They explored the data using *TinkerPlots* graphical tools and techniques that they had learned in the preceding phase of the sequence (see Figure 1 for an example of such); students considered the group differences and what they suggested about the lengths of the two types of fish in the population on the basis of what they observed in this single random sample. The big statistical idea promoted in this lesson was that if a randomly selected sample is assumed to be representative of its parent population, then it can be used as a basis for making claims about that population.

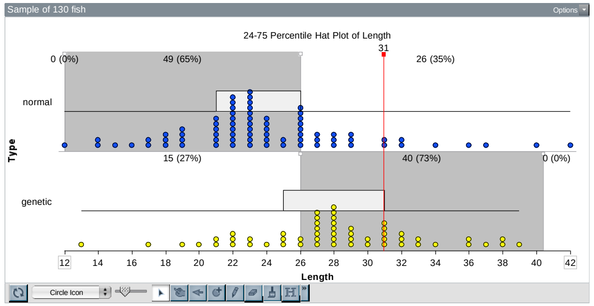


Figure 1: Two students' use of *TinkerPlots* tools to analyze a sample of 130 fish-lengths, separated into lengths of normal fish and genetically engineered fish in the sample.

Lesson 2 The second lesson introduced students to the idea that sampling outcomes are expected to vary from sample to sample, were the sampling process repeated under essentially the same conditions, and that such variability therefore poses a problem for making inferences to an underlying population on the basis of any individual sample. At the same time, the lesson aimed to help students begin to understand that the variability amongst samples exhibits patterns that are predictable over the long run, and that such patterns can be discerned by analyzing collections of sampling outcomes. The lesson began by having students reflect on whether they would expect to obtain similar or different results if another sample of 130 fish were randomly drawn from the fish farmer's pond. Students were then prompted to reflect on whether a sample of 130 fish is big enough to make a confident claim about the sampled population, and to share and justify their intuitions about this question. The lesson then moved to a more systematic exploration of these questions by having students use *TinkerPlots*' sampling simulator to generate several samples of size 130, and then of size 15, from the simulated fish population (see Figure 2). Students recorded the median length of each type of fish in a sample, and the difference between these medians as a measure of the group differences (i.e., difference between the median length of two types of fish in a sample). Figure 2 displays the *TinkerPlots* set-up that the instructor and students used in this investigation; the simulator (left hand tool) used a mixer device to represent the population of mixed fish.

Each selected sample of 130 fish was represented in a case table displaying each fish's type and length. The case table was linked to an ordered and stacked dot plot showing the distribution of lengths of fish in the sample, separated by type and displaying the median of each type as well as the difference between medians using *TinkerPlots*' ruler tool. Each row of the table at the bottom of Figure 2 recorded the value of the three measures for a sample (median length of each type of fish and the difference between those medians) generated by running the simulation once. The four representations displayed in Figure 2 were dynamically linked and automatically updated with the results of each new repetition of the simulated sampling experiment (enacted by pressing the “run” button in the sampler tool). As such, students were able to observe the emergence of the value of the sample statistic (difference between median lengths of the two types of fish) and to track the variability in its values as the repeated sampling process unfolded. Indeed, it was an explicit goal of instruction that students conceptualize the sequential sampling process that began with the population and culminated with the results displayed in the bottom table (and later with a dot plot displaying the resulting distribution of the sample statistic, as shown in Figure 3). This goal was addressed more explicitly in Lesson 3, as described in the next section of the paper.

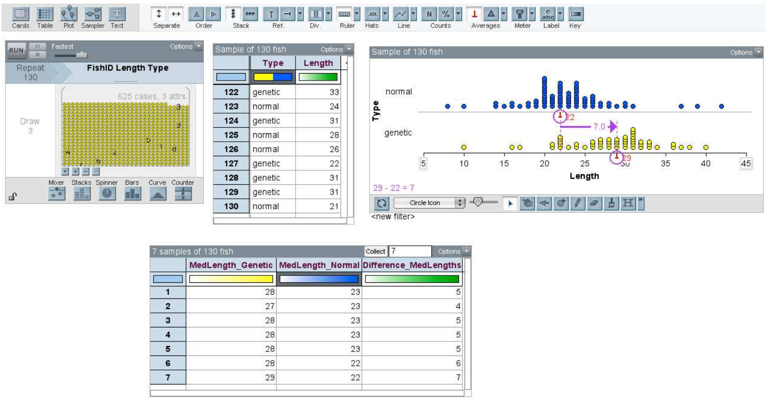
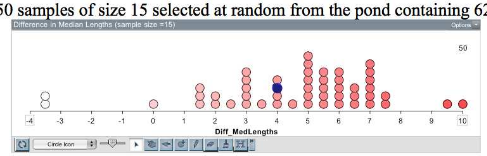


Figure 2: A *TinkerPlots* simulation of sampling 130 fish from a population of 625 mixed fish. The table at the bottom shows the three measures recorded for seven trials of the sampling experiment.

Students explored the patterns in these measures generated by simulating the sampling experiment seven times; they identified similarities and differences among the resulting medians and difference in medians for the collection of seven samples, and they used their observations as a basis for proposing how this might help test the claim that genetically engineered fish tend to grow longer than normal fish in the larger population. Class discussions around this exploration showcased students’ perceived patterns, culminating with a general consensus that the genetically engineered fish in the population were inferred to be “between 4 and 7 centimeters longer” than the normal fish.

Lesson 3 The final lesson built on the activities and issues raised in Lesson 2 by having students examine the effects of sample size on the variability of the difference between median lengths of fish that they had previously explored only for seven samples. The lesson began with a demonstration and discussion of the *TinkerPlots* simulation of selecting 50 samples of size 15 from the simulated fish population (following the set up shown in Figure 2), culminating with the presentation of a distribution of the difference in median lengths for each of the 50 simulated samples as shown in Figure 3.

I. Here is a distribution of the difference between the median lengths of genetic and normal fish for 50 samples of size 15 selected at random from the pond containing 625 fish.



1. Describe what the darkened dot represents:

2. What information is shown by this dot?

3. How was this information obtained? Describe the sequence of steps in the sampling process that produced the information shown by the darkened dot.

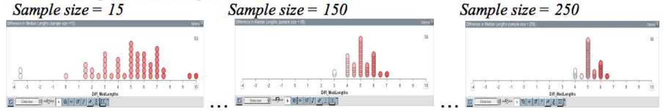
Figure 3: A sampling distribution and accompanying prompts from the opening activity of Lesson 3.

Discussions around this demonstration centered on having students track and explain the process of how the dot plot of the sampling distribution resulted from the sampler in terms of the various intermediate objects produced by the simulation and shown in the *TinkerPlots* window, as displayed in Figure 2. This activity aimed to help students build and solidify their imagery of the repeated sampling process and their meaning for the resulting sampling distribution displayed in Figure 3. The accompanying activity prompts also assessed the strength and robustness of students’ imagery by having them work backwards from a particular point on the dot plot and explain what it represented and the process that produced it. The activity prepared students for the subsequent part of Lesson 3, which required that they be able to decode and interpret a sequence of such dot plots coherently.

In the second part of Lesson 3 students examined and interpreted a sequence of five distributions of the difference in median lengths, each for 50 simulated samples of a different size drawn from the fish population. Students examined and interpreted these sampling distributions in relation to the increases in sample size. A subset of these distributions and the accompanying activity prompts are displayed Figure 4.

II. Here is a sequence of distributions. Each one is of the difference between the median length of genetic and median length of normal fish for 50 samples of a given size selected at random from the pond containing 625 fish.

Sample size = 15 … *Sample size = 150* … *Sample size = 250*



4. Compare these distributions for the various sample sizes. What do you notice about these distributions as sample size increases?

5. What is a big enough sample of fish for the farmer to pick from the pond in order to confidently test the company’s claim that genetic fish tend to grow longer than normal fish? Please explain why you think this.

6. Estimate how much longer the genetic fish in the pond tend to be than the normal fish. How confident are you about this estimate? Please explain.

Figure 4: Prompts and a subset of the distributions from the final activity of Lesson 3.

The first prompt (Question 4) aimed to orient students' attention to the fact that the clustering of the sample statistic becomes increasingly compact (its variability decreases) with increasing sample size. A group discussion of this observation ensued which involved eliciting students' ideas about how to describe and measure the pattern of observed variability. This discussion was followed by prompting students to use this pattern as a basis for choosing a sufficiently large sample in order to confidently infer whether genetically engineered fish tend to grow longer than normal fish in the population (Question 5). The final prompt (Question 6) asked students to estimate *how much longer* genetic fish tend to grow than normal fish. These questions culminated in a group discussion about the trade-off between the competing interests of maximizing sampling accuracy and minimizing sample size.

Three days after Lesson 3, students took an exam designed to assess their thinking and conceptions of the ideas addressed in the instructional sequence. Students responded to a set of questions nearly identical to those from Lesson 3, but couched in a different story context (i.e., testing whether a genetically modified variety of cucumbers tended to grow longer than a normal variety). Follow-up interviews conducted within a week of the exam queried students' responses to this task.

SUMMARY OF SELECTED FINDINGS AND CONCLUSION

Two salient findings regarding students' thinking were revealed in an initial analysis of their responses to the activity prompts of Lessons 1 and 2, and the classroom discussions that emerged around them:

- Class discussions proved to be productive vehicles for helping students notice and reflect on key ideas targeted in the instructional sequence. The whole-group discussions around the orienting prompts and reflection questions of the activity sequence turned out to consistently provide rich opportunities for students to notice patterns of dispersion in the distributions that resulted from re-sampling simulations. As a particular case in point, the discussions around the activity in Lesson 2 (see Figure 2) highlighted several students' observations (derived from the patterns they observed in the bottom table of Figure 2) that "the samples [median lengths] don't vary by much" and that the median length of genetically engineered fish in a sample was larger than that of the normal fish in every sample chosen. This last was taken as evidence, and led to a consensus among students, that the genetically engineered fish in the population tended to grow longer (by between 4 and 7 centimeters) than normal fish. These discussions thus evidenced students' abilities to coordinate two things: 1) their expectation that the sample statistic's value will vary among samples, and 2) their emerging understanding that such variability is not haphazard but is instead constrained to a (possibly) predictable range of values.
- Students generally exhibited an appreciation of two seemingly competing ideas: although an individual random sample can be used to make an inference about the sampled population, repeating a sampling experiment shows that sampling outcomes (i.e., a statistic's values) vary from sample to sample. Significantly, a majority of the students seemed to readily appropriate the idea (promoted in instruction) of using a distribution of multiple values of the sample statistic generated for samples of a particular size as a basis for informally assessing their level of confidence in their inference about the population. For instance, most students asserted that they were "completely" confident in their inference that the genetically modified species of fish were generally between 4 and 7 centimeters longer than normal fish because the value of the statistic of interest invariably fell between those values in all trials of the sampling experiment.

The classroom discussions around the tasks displayed in Figure 4 (exploring the variability across distributions in relation to increasing sample size), together with students' written responses to them and the analogous questions presented in the exam, generally indicated the following:

- Students saw clearly that the variability across the sequence of distributions decreased ("is more condensed") as the sample size increased. Moreover, Question 5 provoked an appreciation among several students of a tension between the competing interests of choosing a relatively small sample and still making a confident inference about the population on the basis of the sampling distribution for samples of that size. These students argued for a trade-off between a big enough sample size and confidence in their inference that was grounded in the following logic: the variability of the sample statistic for samples of the next higher size, as displayed by the range of the sampling distribution, did not decrease appreciably relative to that for the preceding smaller sample size and therefore did not yield an appreciably more "accurate" inference about the population parameter. That is, these students were sufficiently confident in their inference based on a distribution of the statistic for a particular sample size that they thought it not worth the slight increase in confidence incurred by selecting a substantially larger sample. This is exemplified by the following student's response to Question 5:

  S2: 150 fish. The data is very condensed and gives an accurate representation of the full data set. I would not choose 250 because the difference in variability is so menial that the extra work would be pointless and it would not do much to effect your decision.

Students' written responses to the first activity prompts of Lesson 3 (Figure 3), and their explanations of the responses to the analogous task provided in the exit interviews, indicated that many students experienced the following challenges:

- Difficulty understanding what the distributions of the sample statistic (difference between medians lengths) represented in terms of the scenario(s) in which they were embedded. Although most (12) students were able to correctly explain that an individual point in the distribution shown in Figure 3 (Question I.1) represented a particular sample of 15 fish, only 8 of those students had a clear sense of the full information convey by that point. A common difficulty amongst those having an unclear sense involved not seeing that a point indicated the *difference in median lengths of the two types of fish* in a sample, and instead thinking that it showed either an actual length or the median length of fish. Such confusion and other difficulties were seen even among some of those students who evidently understood a point to represent a sample of 15 fish. The following responses to Questions I.2 and I.3 given by one such student illustrate the tendency to interpret the information indicated by the darkened point (see Figure 3) as an average of the 50 data values, rather than thinking of the relation between the point and the sampling process that produced it:

S8: The information shown by this dot is that all of the data added together and divided by 50 is 4" and "This information was obtained by the position of the dot and the information around the dot and from the graph.

The above response is in contrast to one of the most coherent responses to the same questions, as illustrated below:

S9: For this sample, the median of the genetic fish was 4 cm higher than the median of the normal fish." and "A sample of 15 fish was retrieved by the mixer. The program found the medians of the normal and genetic fish and then found the difference between them, which it represented with a dot.

S9's response clearly relates the darkened point to the sampling process that produced it, suggesting her having a vivid imagery of that process and an understanding of how the process generated the information represented by the point. Moreover, her identification of the sample statistic as a difference between the median lengths of two subsets of the sample suggests her ability to parse and hold the three quantities in mind as distinct entities without confounding them. All of this is indicative of an ability to keep mental track of, and describe, the multi-tiered simulated re-sampling process (i.e., conceiving of the coordinated sequence of actions and resulting statistical objects) that begins with an identifiable simulated population, involves a random sampling process, and culminates in the generation of a distribution of the appropriate sample statistic. Relatively few of the students evidenced having developed this ability to the same extent as S8.

These findings underscore challenges in designing instruction to foster the development of (informal) distribution-based inferential reasoning, and they orient us to consider possible refinements to the instructional activities. One possible refinement is inspired by Harel's repeated reasoning instructional principle (Harel & Koichu, 2010). The refinement would elaborate the activity shown in Figure 3 by having students practice imagining and explicitly identifying and distinguishing the processes and intermediate objects generated by the *TinkerPlots* re-sampling simulation that produces the distributions of the sample statistic shown in Figures 3 and 4. Such an elaboration would aim to help students build stable and robust mental images of the re-sampling scheme and its products, thereby supporting their ability to hold the ensemble of coordinated ideas in mind as they attempt to make distribution-based inferences and quantify their confidence in them.

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PEDAGOGICAL STATISTICAL KNOWLEDGE OF A PROSPECTIVE TEACHER

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We present a case study of a prospective kindergarten and elementary teacher at the end of her teacher education program, when she had the opportunity to teach statistics in a classroom. With the aim of understanding her pedagogical statistical knowledge, we focus on her knowledge of students and knowledge of teaching. Results show that this prospective teacher reveals some knowledge of students and of teaching methods and strategies, but seems to have difficulty in translating that to practice.

INTRODUCTION

In a world dominated by numbers, teaching statistics is crucial to create citizens able to critique the information around them, knowing how to deal rationally with that information (Fabrizio, López, & Plencovich, 2007). To teach statistics, teachers need to know how to organize data, to interpret it, to build knowledge from it and to know when it is appropriate to do so (Batanero, Godino, & Roa, 2004). In addition to this statistical knowledge, pedagogical statistical knowledge is also required. As Batanero et al. (2004) state, teachers need to know how to help their students to develop correct intuitions in this field and how to deal with controversial ideas. Therefore, teacher education programs have an important role in developing both prospective teachers' statistical knowledge and pedagogical statistical knowledge. However, little research has being made to determine which knowledge teachers have and which is necessary to teach statistics (Chick & Pierce, 2008). In this paper we strive to understand the pedagogical statistical knowledge of a prospective kindergarten and elementary teacher (grades 1-4), focusing on her knowledge of students and knowledge of teaching.

CONCEPTUAL FRAMEWORK

Since Shulman's work (1986) that teachers' knowledge has been categorized as content knowledge and pedagogical content knowledge. Together with a sound statistical knowledge, teachers need pedagogical statistical knowledge to be able to effectively teach their students (Shulman, 1986). Sorto and White (2004) highlight knowledge of students and knowledge of teaching as key aspects of pedagogical statistical knowledge. In their view, knowledge of students involves the knowledge of conceptions and misconceptions, learning processes, students' work and classroom discourse. In addition, it includes the ability to interpret students' responses and to analyze their strategies and solutions in order to infer about their understanding. Batanero et al. (2004) also mention the ability to predict students' learning difficulties, mistakes, obstacles and problem-solving strategies. Studies that focus on teachers' knowledge of students indicate that they know little about students' statistical knowledge, their difficulties and ways to overcome them. Reporting on a study of 42 teachers, Watson, Callingham, and Donne

(2008) conclude that some of them make mistakes on items that call for a response to a particular misunderstanding of a student and cannot identify the next steps to take students to develop statistical understanding. Nicholson and Darnton (2003) also report that teachers are not familiar with students' common difficulties and misconceptions. González and Pinto (2008) conducted an investigation with four prospective teachers on statistical concepts of data organization and representation and concluded that they had no idea of students' difficulties on this topic. Consequently, they missed opportunities to help their students to confront these difficulties and achieve a deeper understanding of fundamental concepts.

Regarding the knowledge of teaching, Sorto and White (2004) report that teachers must have knowledge of teaching methods and strategies, explanations and didactic resources. For example, teachers should be able to recognize and take advantage of statistical everyday situations and use them to teach (Chick & Pierce, 2008). As for specific methods to teach this subject, emphasis is given in conducting experiments in the classroom, particularly in groups, to develop the understanding of certain concepts and linkages with other areas of the curriculum, taking into account students' interests and abilities (Ponte & Fonseca, 2001). Additionally, the GAISE report (Franklin et al., 2007) recommends that teachers: (i) emphasize the development of statistical literacy and statistical thinking and reasoning; (ii) use real and motivating data; (iii) emphasize the understanding of statistical concepts, as opposed to only teach theory and procedures; (iv) foster students' active role in the learning process; (v) make use of technology; and (vi) use assessment as a resource for improving teaching and learning. Concerning the teaching of graphs, Curcio's levels of graphs comprehension (1987) (read the data, read between the data and read beyond the data) are fundamental in teachers' knowledge to foster students' statistical literacy. Researching prospective teachers' difficulties regarding knowledge of teaching, Fernandes (2009) indicates that they have trouble in planning lessons and managing unforeseen situations and time. This author mentions that usually they adopt a more traditional approach (presentation of concepts, centered in the teacher, and practice of exercises). A study of Chick and Pierce (2008) shows that most of the prospective elementary teachers' lesson plans focus on correct graphs, rules of presentation and calculation of measures, but put little emphasis on understanding data sources and its implications. In another investigation, González and Pinto (2008) conclude that prospective teachers have no idea about the levels of understanding of statistical graphs.

METHODOLOGY

This study follows the interpretative paradigm and a qualitative approach, using a case study design with multiple data collection instruments: questionnaire, interviews, observations and document collection. Data analysis is descriptive and interpretative. The participant is a prospective kindergarten and elementary teacher (grades 1-4), with the fictitious name Dora, selected for being in the last years of a teacher education program with the possibility to teach statistics during her internship. In her 2nd year of studies (in 2010/11), she took the single course of the teacher education program dedicated to statistics, given by the first author (who was also responsible for the data collection). During the course, prospective teachers worked statistical concepts through the exploration of situations in real contexts, with emphasis on representing and organizing data and statistical measures and their interpretation, and discussed how to pursue statistical investigations using different data collection instruments. In groups of up to three prospective teachers, they carried out a full statistical investigation, spending around 10 weeks. During her 3rd year of studies, Dora answered a questionnaire with content regarding statistics and didactics of statistics, being encouraged to present her line of thought. Her answers to this questionnaire (Q*x*) were afterwards discussed during interviews. During the 2nd semester of the 4th year (in 2012/13), she was enrolled in a supervised teaching course, having to teach a few days to a grade 2 class. Aside from an initial (EI) and a final interview (EF), Dora was asked to implement a statistical investigation with her students, which she divided in two different classes (in two consecutive days). Those classes were observed and video recorded and interviews were made before (EA) and after (ED) each class to discuss the plan and to reflect about what happened in class. Her reflections of the classes she taught (DB) were also collected.

RESULTS AND DISCUSSION

Dora is a very motivated and committed. She states that "up to the 9th grade [she] didn't like [mathematics]" (EI), because "it was always the same. Always traditional. [The teacher] explained and then we did exercises" (EF). She mentions that this changed when, in high school, she decided to follow the area of Social and Human Sciences and the work "was more practical" (EI). Her enthusiasm for statistics is evidenced by the choice of this theme for her masters' final report. She believes "it is very funny to work data organization and analysis with [children]" (EI) and says that she intends to "check which were the difficulties that they had, not only in the realization of graphs but also in their interpretation" (EF).

Knowledge of students

We discuss Dora's ability to predict students' difficulties, identify their mistakes and suggest ways to overcome these mistakes and her knowledge of students' different strategies. With regard to anticipating students' difficulties, in the questionnaire she had to build a frequency table and a plot for a data set and to formulate conclusions based on those representations. Later, she was asked to identify possible difficulties for 6th grade students in solving the previous tasks:

> I believe that it may be at the level of the graph's findings and in the construction of the frequency table, since the results are scattered, and thus there is the need to seek and count all equal results, which requires some concentration. (Q1.4)

We note that Dora identifies as difficult the data management (in the table construction or in the formulation of conclusions) since there are many and different data. When she had to justify her response, she stated "[students] could forget certain data" (EI). She considers that it will be difficult for students to organize and present

data, but doesn't point out as difficult for them, for example, to draw conclusions that are more than a mere description of explicit data on representation.

During the statistical investigation that she conducted in the classroom, Dora took students to build a graph on which she wanted them to mark on the vertical axis their weights. Talking about students' potential difficulties in doing this, she said "they will have a hard time making the graph, because of the scales" (EA). She did not point this difficulty in the previous situation since, possibly, she thought that this is no longer an obstacle for 6th graders. In fact, some of her 2nd graders showed problems in this direction, both in the choice of unit and in the correct marking of axis values:

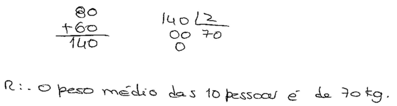
> Some students used the squares on the sheet of paper and placed the scale numbers inside, with no space for data that were in the range between two numbers (between 20 and 25, for example) (...) The bars of these two values became equal [20 and 22], since they couldn't score data in the intervals of two numbers. (DB)

To address the difficulty of incorrect marking of the axis values, Dora decided that she should "convey some of that information so that groups could improve graphs" (DB). In fact, in the next day, she presented students "an example of another group" to discuss "what was the most perceptible and what was the most useful" (EF). In this situation, she decided to explain the correct way to make the graph with reference to one properly built. To lead students to overcome the obstacle of not choosing a constant unit on the vertical axis she said:

> Dora: Now you have to keep in mind here in this side. That this here cannot be as it is in here. 1, 3 and then jump into the 7 and the 9; it must be a regular thing. How often can you make it? It's like a sequence (...) If you are making 1 to 1, you cannot bring it up to... What is your maximum? (...)
>
> Student: 29 (...)
>
> Student: From 2 to 2.
>
> Dora: From 2 to 2 or...
>
> Student: 5 to 5. (Class)

Dora shows the need to explain how to properly mark the axis scale, inducing the unit 5. After reflecting on her lesson, she stated that students "should have had the opportunity to explore the situations mentioned [like using a scale with the unit 2], but due to lack of time this didn't occur and I even had to induce them to this scale [of unit 5]" (DB). In this case, she takes control in overcoming students' difficulties, using the strategy of explaining the correct procedure, which she explained by time constraints.

With regard to Dora's ability to identify students' errors and to suggest strategies to help students to overcome them, we showed her the following problem "There are 10 people in an elevator, 4 women and 6 men. The average weight of the women is 60 kg and the average weight of men is 80 kg. What is the average weight of the 10 people?" and a student's solution showing a common misconception (figure 1).



R: The mean weight of the 10 people is 70kg

Figure 1: Student's response to a problem involving the concept of mean.

In the questionnaire, Dora wrote "I can't answer" (Q4.3.1), and this issue was brought up during one of the interviews:

Dora: Ah, I can see now. He joined the mean of the 2 and divided by 2.

Researcher: And why is it wrong?

Dora: For me this is wrong because the 2 doesn't represent the total number of people.

Researcher: So how should the student have done?

Dora: (...) So, it should be divided by 10.

Researcher: So, is it 14 kg?

Dora: Well, I don't know. No, but he doesn't know the total number of... But is this really wrong? (...) For me it was easier to have the data of all men and women and divide by the number of people (...) There's not enough data (...) I don't know if this is possible to solve. (EI)

The difficulty of Dora in this matter is linked to the fact that she did not know what would be the correct answer and therefore an appropriate strategy (Q4.3.2), showing a strong link between content and pedagogical content knowledge. Some lack of understanding on the mean interferes with her pedagogical statistical knowledge, making it difficult to her to present strategies to help the student in this problem.

Student 1: Student 2:

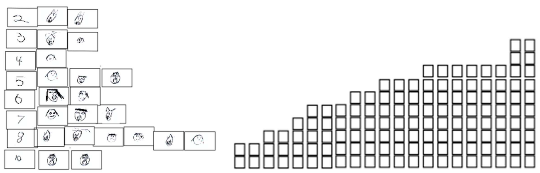


Figure 2: Representation of two students for the same data set.

Regarding the interpretation of students' strategies, Dora was shown two different representations, carried out by 2nd graders, depicting the number of teeth that each classmate had already lost (figure 2), and was asked to compare them. As a difference between them, Dora pointed that "the student 1 made the representation horizontally, while the student 2 made it vertically" and she saw no similarities (Q6.1). During the interview, she was asked again to explain these representations, and answered:

> 2 [students] lost 2 [teeth], 3 [students] lost 2 [teeth], 4 [students] lost 1 [tooth], [looking at student 1 representation] (...) I cannot remember how this [student 2] one here did (...) I don't know... because I know that this [representation of student 2] should show the same

> as this one [representation of student 1]. [She turns the sheet in the opposite direction to realize if the representations have the same visual aspect] (...) But I think that to 2 students fell 10 [teeth] [looking to the last two columns of the student 2]. Oh, that's right! [looking to the representation of student 1], 2 students fell 10 [teeth]... That's it. (EI)

We observe that Dora began by explaining the representation of student 1, assuming that the numbers that this student wrote were the frequency and the cards with drawings in front of each number were the number of teeth. When she tried to explain the representation of student 2, Dora seemed to realize that her understanding of the first student's representation was not quite right, since they should represent the same information. When looking at the last two columns of the representation of student 2, she concluded that each column in this case is the number of teeth lost by each student and that only thinking in this way the two representations conveyed the same information. However, faced with the question of which representation she thought to be more complex, she chose the representation of student 2, since initially "I didn't understand it. I needed to ask him (...) Now I get it, because I have this [representation of student 1] right here" (EI). Although she only properly understood the representation of student 1 after analyzing the representation of student 2, she continued to find that the representation of student 2 was more complex. This demonstrates that she associates the complexity of students' representations to the difficulty or not in interpreting them. Another reason may be due to the link of the cards' drawings of student 1 to a more childish representation.

Knowledge of teaching

We also discuss the different teaching methods and strategies Dora used during the classes that she taught or that she indicated to follow. In the statistical investigation that she conducted with her 2nd graders, it was discussed students' gain of weight. Dora considered that students can and should assume control at different phases of this process: "early in the investigation, I [should] give them room to talk. They were a little anxious and nervous. And they were very restless, then that also eventually led me to be controlling" (EFD). Nevertheless, in her classroom, she did not allow for much independence from students during the investigation, namely in posing a problem, making a plan and choosing representations (we show this in detail in Santos & Ponte, 2014).

In what concerns the type of work with students, Dora adopted several times group work. She showed knowledge of students in the constitution of groups since her decisions were very productive and efficient, according to her supervisor teacher. When elaborating groups she took "in consideration the existence of a leader, to help to organize work and that requires some elements to follow him/her" (DB). Yet, management of group work was one of her main concerns:

> For example, tomorrow I think will be very hard to manage all groups and help them (...) If they were used to this, to this type of work, maybe it wouldn't be so difficult. Now, I know this will be the first time they will do work like this (EI).

Dora was very worried about the independent work of the different groups, considering that students were not used to work on open tasks. She was also concerned with her role in the classroom, since she did not have experienced this type of work as student: "I never did something like this. I never did a statistical investigation with them (...) It seems different (...) To teach other things, they were things I learned in school and that they did with me. Now this, no" (EI). However, in class, her activity was impressive, being aware and following each group work. She also shows that to teach statistics it is important for students to be main actors of their own learning: "Firstly I think it should always come from them, which means, do statistics with them (...) Which is, for them to collect data, organize it in tables or as they want, then construct graphs and, then, from there, take conclusions" (EF).

Concerning teaching strategies to work the analysis of graphs, Dora did some research on her own. Besides showing the perspective of the importance of elaborating hypothesis during the construction of graphs, during the analysis of a pictogram in the classroom, she encouraged her students to examine the representation, which may allow them to confirm or contradict their initial hypotheses:

> I started with the question "What can be said about this graph?", since it's a very open question and allows a diversity of answers, that sometimes we wouldn't have access if we were only to ask the mode, minimum, what data have the same values, etc. (...) To conclude the analysis of the pictogram I placed another question to the group: "How many more boys like snakes than cats?" (DB).

First she places an open question to see what the focus of different students is, and then she poses a question that requires a reading between the data, to go beyond the literal reading of information. During the analysis of the graph built during the weights investigation that she led in classroom, she also took into account the literature she read about questioning, such as the connections with other mathematical concepts, like the case of the estimation, and the attempt to break some stereotypes regarding weight, speaking of students' height: "This work was very interesting, especially when we held it in a large group, because I think I made an interesting exploration of the graph, touching in key aspects such as comparison between weights and heights and estimation" (DB).

In the statistical investigation that she conducted with students, Dora considered that the new topics for them were the formal aspects of graphing (title, scales, axis, labels) Initially, she had no clear idea about how to introduce this: "I also don't know very well" (EA). Dora predicted some difficulties regarding the scales, where she controlled the discussion, taking students to adopt the unit of measure she considered appropriate. She justified this type of classroom decisions at certain moments by saying that it is important to take students to get to the most useful methods: "Besides being the students to suggest the data organization, the intern's role will always be to take the group to think if that is the most advantage record method" (DB). To discuss the importance of the graph's title, she prepared two graphs "one with nothing and

the other with that kind of information for them to verify which one gives the most complete information" (EA) and took students to compare the information on them:

Dora:	I have here these two graphs (...) Which one do you think it is best represented? Most perceptive?
Student:	This is wrong, isn't it? Because I'm not the heaviest.
Dora:	No, this isn't the weight (...) Do you know, from this graph, what is it about?
Student:	No.
Dora:	And from this? Can you see?
Student:	It's the height.
Dora:	Is this the height? What does it say here?
Student:	Number of times that goes to the movies each month. You go more times to the movies.
Dora:	From this you already know what it is (...) Put a title.
Student:	Title?
Dora:	Yes, it should have a title so that we know what it is about. (Class)

In this classroom excerpt, we observe that to introduce the graph conventions, Dora used the explanation and the supply of properly constructed examples for students to consider the advantages of presenting this information. In another situation proposed in an interview, she had to plan the introduction of the concept of mode to a group of students. Dora said that she "could grab these materials [math cubes] and began by asking which is the most prevalent color in this set. Then from there introduce the mode concept" (EF). Thus, to present this concept she thought of a task with manipulative materials. It should also be noted that she did not consider explaining the concept first and then applying it. On the contrary, she seemed to initiate the application of the concept and only then introduced the proper denomination. When asked to indicate the difference between concepts being introduced through a task or through its emergence during a statistical investigation, she said:

> It's different. The fact that sometimes it arises spontaneously turns out to work better, on the experience I have (...) If I took a task already made to work the mode... It's different. I can't explain (...) Also because the context there would be different. We would already be talking about other statistical concepts; here we would only be facing the mode concept (...) I don't know if it would be better, or if it would be worse. It would be different (...) And depends on the children's experience (...) It's just different (EF).

Thus, Dora sees many advantages in using statistical investigations in the classroom, but indicates that it is not necessary to perform an investigation whenever she wants to introduce a new concept. However, we observe that she gives great importance to teaching statistics through investigations, not only to learn concepts, but also for other reasons:

Through these investigations, I think they end up gaining much more not only regarding their mathematical learning, but also from other areas, especially at the personal and social level. Since they are in groups, they have to listen, know how to be heard, respect others' ideas and that area ends up to be developed and worked (EF).

CONCLUSION

About Dora's pedagogical statistical knowledge, we conclude that she has some knowledge of students, predicting correctly some difficulties in problematic situations (elaborate the graph scale, organize and present data), contradicting Nicholson and Darnton's findings (2003). To help students overcome their obstacles, she decides to explain the suitable procedures or, sometimes, compares it to a more appropriate solution. However, she demonstrates that she cannot discern the complexity of different data representations. These difficulties may be associated with shallow knowledge or no understanding of certain statistical concepts and procedures, and are thus connected with some flaws in statistical knowledge.

This prospective teacher demonstrates to know and make use of teaching strategies and methods recommended by several authors (Curcio, 1987; Franklin et al., 2007; Ponte & Fonseca, 2001): group work, open ended tasks, statistical investigations, use of real data, different levels of graph analysis. This was, probably, due to her special interest in the theme and her study of the literature. In class, sometimes, she ends up adopting a more expository teaching, explaining correct procedures (construction accurately of a graph with reference to the title and scale), which she mainly justifies by time constraints, as pointed by Fernandes (2009). Some difficulties that she points are mainly due to lack of experience as a student with this kind of work.

To Groth (2007), it is essential that the learning of statistics and its didactics, by prospective teachers, do not occur separately, but in the same course. Thus, the ideal would be that opportunities, during the teacher education program, for open ended tasks and for teaching methods to develop an understanding of statistical concepts could help to increase prospective teachers' experience as students, but also serve as discussion moments of what is required to conduct this kind of work in their future with students. During the internship, this work has to be continued, with moments of preparation and reflection about classes. Possibly, this is a job that must be continued during the induction period and even afterwards, given the complexity involved in the construction of knowledge of all topics and its didactics.

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INTERPRETATION OF PICTOGRAMS BY 3rd GRADE PUPILS: THE TEACHER'S ROLE

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In this paper we analyze how a primary school teacher (João) leads the discussion of a task in a grade 3 class aimed at reading and interpreting pictograms. The paper is part of a larger study that follows an interpretative and qualitative research methodology with a case study design. The results indicate that, in his teaching practice, the teacher strives to ask questions so that his pupils are not limited to the identification of categories of high and low frequency characteristic, that is, "reading the data", but also raises questions related to "reading between the data" and "beyond the data."

INTRODUCTION

Statistics is a part of global education, helping future citizens to obtain reading and interpretation skills of tables and graphs that appear daily in the media and fostering the development of critical thinking based on data. Martins and Ponte (2010) advocate the inclusion of statistical education since the elementary levels of education contributing to the development of a critical and participative citizenship. Although the Statistics curriculum of many countries includes the teaching of graphics as a topic since the beginning of education, the pedagogical emphasis is aimed more at aspects relating to construction rather than the interpretation of graphs (Monteiro & Ainley, 2010). In this paper, we intend to examine how the questions posed by a 3rd grade teacher in his class promote reading and interpreting of pictograms by pupils.

READING AND INTERPRETATION OF STATISTICAL GRAPHS

The link between school and everyday life can be achieved by statistics taking advantage of the presence of various types of data in the media (Arteaga, Batanero, & Contreras, 2011). According to these authors, one of the components of statistical literacy we need to develop in pupils is the ability to interpret and critically evaluate statistical information represented in graphs, as this also constitutes an important part of statistical culture.

Graphs comprehension can be defined as the ability to derive meanings from graphs created by others, or by themselves (Friel, Curcio, & Bright, 2001). Reading and interpreting graphs requires knowledge of their structural elements and their conventions - such as title, labels, axes, scales, represented variables and figurative elements (lines, points, bars). It also requires an integrated reading and interpretation of these structural elements.

The skills related to reading and understanding of statistical graphs have been studied by various authors and the classification proposed by Curcio (1989) is the one that had the greatest impact on statistical education. It includes the following levels:

- Reading the data: direct reading of the graph without interpreting the information contained. Pupils should just respond to direct questions.
- Reading between the data: interpretation and integration of data in the graphs. This capability requires the comparison of the data or the identification mathematical relationships;
- Reading beyond the data: this includes the organization of predictions and inferences based on the interpretation of data on information that is not directly reflected in the graph.

In addition to the interpretation of graphs, Aoyama's classification (2007) considers a critical evaluation component of the information contained in the graph, reflecting on the following levels:

- Rational/literal level: Pupils can correctly read the graphs and detect tendencies but they do neither question the information nor suggest alternative explanations. They identify the relationship between variables but they do not try to explain it.
- Critical level: Pupils read the graphs, understand the context and assess the information reliability, sometimes questioning the information presented.
- Hypothetical level: Pupils read the graphs, interpret and evaluate the information to create their own explanatory hypotheses.

González, Espinel, and Ainley (2011) suggest the idea of graphical competence including (i) the ability to extract information from different types of graphs and to interpret meanings from reading the data; (ii) the ability to select and create graphs suitable for specific situations, with or without the support of technology; and (iii) the ability to critically assess graphs and to distinguish the strengths and limitations of certain graphs. From their perspective, teachers are faced with the challenge of developing their analytical competence of graphs and mastering the required knowledge for their accomplishment in the classroom, as this is an important purpose of Statistics' teaching.

THE TEACHER QUESTIONING

The participation and involvement of pupils in the entire data processing process are key aspects in the interpretation of the graphs to which the teacher should be particularly alert to. Friel, Curcio, and Bright (2001) point out that asking questions is closely connected with the understanding and that "teachers need to develop a framework to help them think of questions to be asked. This question-asking framework is important for the understanding of graphs" (pp. 129-130). To these authors, different levels of questions promote different levels of graphical understanding, so the teacher should pose questions that are not limited to extracting data from graphs; but they should also allow for the identification of relationships as well as prediction and generalization, thus fostering development of the ability to understand graphs.

Friel, Curcio, and Bright (2001) define three levels of questions that the teacher should use to foster the graphical understanding levels considered by Curcio (1989). The elementary level questions require information gathering to respond to immediate questions which can be easily identified in the graph. At the intermediate level, questions aim at integration and interpretation of the available information in the graph and the search for relationships between data. Finally, the advanced level questions require extrapolation of the data and the analysis of the implicit relationships in the graph. Although Friel, Bright, and Curcio (2001) believe the advanced level questions, allowing to "read beyond the data", are the most challenging and contribute to a deeper understanding of the data structure, they believe that it is necessary to contemplate the three types of questions to promote the process of graphical comprehension.

METHODOLOGY

This study is part of a broader research work in a collaborative work environment, involving the first author and three teachers from the 3rd and 4th grades, with the purpose analyzing their professional practices for teaching organization and handling of data. Working sessions include preparation of tasks and the discussion and reflection of their exploration in the classroom. In these sessions, the researcher streamlines sessions; he helps with the preparation of the tasks and with the reflection over their accomplishment. The collaborative group decided to work on investigative tasks relating to the daily lives of pupils, involving the formulation of research questions, collection and organization of data, presentation and justification of conclusions (Martins & Ponte, 2010).

We have followed a qualitative research methodology of interpretative nature, using the case study method (Stake, 2007). In this paper, we intend to examine how the questions raised by João (one of the case studies) promote the reading and interpretation of pictograms by pupils. Originally, this teacher has a degree in Primary Teaching, and he completed his training in the Universidade Aberta (Open University) as a Portuguese Language major. At the beginning of the study, he had been teaching for 33 years.

Data were collected through classroom observation, with video and audio recording, supplemented with interviews, participation in workshops and collection of materials produced by pupils. Data analysis is based on three levels of questions for graphical understanding proposed by Friel, Curcio, and Bright (2001).

THE TASK "SCHOOL PUPILS' TELEVISION PREFERENCES"

In the initial class [January 29. 2013] teacher and pupils decided to ask the following question for study: "What are our favourite TV programs?" After some discussion, they wrote on the board the name of the type of television programs to be voted

(documentaries, cartoons, soap operas, news, movies and sports) and they choose their favourite programs. The data was placed in a table which contained the name of each pupil. They created a table of frequencies and a pictogram and they formulated conclusions with reference to the analysis of the collected data. From the discussion conducted, came the idea of extending the study to other classes in order to find out the preferences of their colleagues. At a later class, teacher and students planed the data collection work in each class to carry out this study.

Preparation

Collection and analysis of pupils' television preferences. Pupils organized themselves in groups and went to other classrooms to explain to their colleagues the work they were conducting and what they wanted to find out about their television preferences. They distributed a ballot and explained that they should just put a cross on the type of favourite television programs and another cross on gender to differentiate the preferences of boys and girls. Before the vote, they also explained what they meant by each of the types of program. After the voting, the ballots were collected, put in an envelope and brought to their class for organization and processing of data.

In class of March 2, 2013, teachers and pupils recollected the work they did and identified the steps for continuing the study. They started by counting the data and organized it into frequency tables, separated by boys and girls with the respective graphical representation. As they had done for their class, they would be building pictograms, allowing for comparisons with the choices from their classmates. To finish the work, the pupils should be drawing conclusions for each class. We should note the teacher's reminder to perform a careful reading of the graphic so as to "discover interesting things". The various groups received the envelope with the ballots from each class; they performed a counting of the choices and built a frequency table. They got a paperboard to create the pictogram and they proceeded to make the respective measurements of the space to be occupied by each data category, making sure to keep them separated from one another. In order to represent the choice of each pupil, they used a square of paper with an image [smiley face]. They painted the square blue if the data belonged to a boy and pink for girls. They glued them and put a legend on each category and they started creating designs that facilitate their identification [Appendix 1 and 2].

Formulation of conclusions. After creating the pictograms, each group analyzed their data and started to formulate conclusions. The teacher went around through the groups asking them to read texts produced and corrected writing and grammar aspects in sentences. His interventions were aimed at helping pupils to formulate conclusions. He was worried about the pupils analyzing the information provided by pictogram in order to answer the study question. In addition, he was asking other questions to challenge pupils with the intent of getting them to identify other more "interesting" aspects represented by the pictogram, enabling comparisons and relationships between values expressed by the various categories. Finally, he suggested that the text was organized according to a certain sequence of sentences.

Interpretation of results

Beginning of discussion. After the formulation of conclusions, the teacher asked pupils to distribute the paperboards with the respective pictograms though several areas of the room so that would be visible to everyone. The following is a presentation of the findings from each group, with a pupil doing the reading while another pupil indicates in the pictogram the data referring to the findings. The following paragraph refers to the presentation of the conclusions class AC from the 2nd grade:

> The type of program chosen the most was cartoons. The second most chosen program was sports. The types of programs less chosen were the news and documentaries. They were both chosen by the female gender. It's balanced, the cartoons were chosen by 5 boys and 5 girls. The difference between the cartoons and sports is 4 choices. The soap operas have 6 less choices than cartoons.

The group that dealt with the data from this class began by identifying the key aspects revealed by the pictogram, i.e., programs which were chosen more and less. At this stage pupils were only "reading data". At a second stage, their analysis indicates that the choices of the less preferred programs were made by the female pupils, and then they moved to another type of analysis looking for differences between the choices of the different types of programs. Their references evolve into "reading between the data."

Choices of girls. In the following interventions, after having read the findings for each group, the teacher asks the class to participate in the discussion of the data presented. He began by suggesting that they should look at the pictograms carefully, looking for "interesting things" and he asked several questions to support pupils in their analysis. After the presentation of the conclusions from the AC class from the 1st grade, the following dialog occurred:

Teacher: Look, where are the girls' preferences going?

Pupils: To documentaries.

Teacher: To documentaries. And now look at the pink spots scattered there, where are the other pink spots going?

Pupils: To soap operas.

Teacher: One for the soap operas and...

Pupils: To the news and movies.

Teacher: And to the movies. And how many girls chose the documentaries?

Pupils: 7.

Teacher: Seven. Now then make the sum of the other girls, from the other pink spots.

Pupils: We already know.

Duarte: 4.

Teacher: Four. Who has something to say about this? Here are seven girls (in the documentaries). And then there is one (he is point at the graph), two, three, four. Isn't this interesting?

Pupils: It is.

Teacher: And what's interesting here? What can we say here? Leandro?

Leandro: Girls who have chosen the documentaries in the type of program, they all together make up more than all of the others separated.

Teacher: That's right. So what can we still say? Something else. Fabio, you say it. I don't know if you've heard Leandro. Leandro explained it to me a bit clumsily but he said: seven girls there and the others are only four.

Fabio: Most girls chose the documentaries.

Teacher: Isn't that interesting? A while ago, I told you to open your eyes. Most girls chose...

Pupils: Documentaries.

João tried to get pupils to focus on aspects of the graph that went beyond simply reading the data, identifying more and less chosen programs. In this case, he suggested an observation over the choices of girls from this class. The teacher wants pupils to see that most girls focused their votes in choosing "Documentaries". Seven girls chose this type of program while the other four were scattered over the other programs. This episode revealed the questions of intermediate level asked by the teacher, aiming to support pupils in reading the graphs so as to develop skills which go beyond a "reading of the data", but which allowed them to move to a "reading between the data" - in this case, focused on the distribution of frequencies relating to the choices of girls.

Justification for the choice of documentaries in the 1st grade. During the presentation of the AC class data from the 1st grade, one of the pupils -Leandro - goes with a possible justification for the preference of documentaries by most pupils from this class, the taste for programs on animal wildlife:

Teacher: The debate is open. I'm enjoying the debate. Let's go.

Leandro: In the 1st grade, I think they chose documentaries because they may enjoy the wildlife.

Teacher: Yes, possibly.

Leandro: I like it a lot.

Teacher: And they also like it.

Leandro did not simply note that "documentaries" were the most chosen type of program but he was trying to "read beyond the data". He provided a justification for why the choice did not go to cartoons unlike in almost every other class. Leandro included programs on "wildlife", which he personally prefers in the category "documentaries", thus using information that was not available in the graph. Leandro's intervention followed the teacher's invitation to participate in the debate.

Balanced choices of boys and girls. In the presentation of the AC class from the 2nd grade, pupils identified a relationship between the data for the program "cartoons", by making a comparison between the choices of boys and girls:

Fabio: We mean to say that there are 5 guys here. 5 boys chose cartoons and 5 girls (pointing at the pictogram).

Teacher: Now Ricardo would like to explain this in another way. Go ahead, Ricardo.

Ricardo: In the cartoons, 5 boys is balanced with 5 girls. If we add it together, the boys and the girls is balanced.

Fabio's intervention pointed to the number of choices of boys and girls while Ricardo put forward the notion of "balance" between the number of boys and girls who had chosen cartoons. While Fabio's reading seemed to address the number of choices, Ricardo's intervention went further to identifying a relationship between these values, thus seeking to make a "reading between the data."

Comparison of two groups of the same year. After presenting the data from the two classes of the 2nd grade the teacher challenged pupils to look for both pictograms and compare the choices made by pupils in from those classes:

Bernardo: The soap operas in both classes are...

Teacher: Not the soap operas, the number of choices of soap operas. Go on.

Bernardo: They are the same.

Professor: More, what else is there that's more interesting in terms of comparison, Duarte.

Duarte. Sports choices are the same, in the two, sports are the same.

Teacher: How many choices for sports on the type of program?

Duarte. 6.

Teacher: 6. 3 girls in that one and 3 boys and this one...

Duarte. Only boys.

Teacher: There is only one girl. It's the same (...) What is the big difference then, in terms of choice... from one class to the other, where is the main difference? Say it.

Isabel. In cartoons.

Teacher: It's in the cartoons. We have just seen it just now in terms of comparison, right? In that class a lot of pupils chose the ... (he waits)

Pupils: Cartoons.

Teacher: Cartoons. A part from that, the other programs have more or less the same number of choices.

Isabel: It's similar.

Teacher: It's more or less the same.

Proceeding with the way in which he intended pupils to interpret the data, João continued to challenge pupils to look for "interesting" relationships. In this last episode, he used the comparison of the pictograms from both 2nd grade classes, thus allowing for the "reading beyond the data" and for the identification of common characteristics (similar) and the main difference (i.e., the number of choices of "cartoons"). João asked advanced level questions aiming to help pupils focus on the data that allowed for comparison of the two classes and interpretation of the results. He also suggested this type of analysis to compare the data of the 3rd grade. The data from his own class (3rd AC) were compared with data from the next door class (3rd BC):

Teacher: Look carefully at the two pictograms. Is there anything else in both classes in terms of choices? Isn't there anything else which is similar?

Fabio: It's the movies.

Teacher: The movies?

Duarte: And sports.

Teacher: And sports. So one class and the other class, how many bars that stand out?

Fabio: 3.

Isabel: They're the same.

Teacher: They're the same, that's it. What are the three types of programs chosen the most both in our class and the other class?

Pupils: Cartoons, movies and sports.

Teacher: Isn't that interesting? The three most chosen program types match, don't they?

Pupils: They do.

In the analysis of the pictograms from 3rd grade classes, the teacher again used advanced level questions aimed at checking for similarities between the choices of the two classes. Taking advantage of the "visual spot" of both pictograms, pupils easily identified that the three bars with the most choices corresponded to the same type of programs in both classes (cartoons, movies and sports). In the interpretation of the pictograms relating to of 4th grade classes, the teacher again challenged pupils to look closely at the pictograms, asking advanced level questions, looking for a "reading that goes beyond the data". Pupils conclude that the choices of the classes are very similar, revealing the same preferences, so any graph could represent the choices of any class.

FINAL REMARKS

Throughout the various moments of performing the task, João attributes great importance to clarifying the procedures that pupils must take to continue their work. This concern is reflected in the preparation of data collection for the various classes, in the identification of the next steps in the task, and in preparation the conclusions.

The discussion promoted by João showed concern in engaging pupils in interpreting the data represented in the various pictograms. A representative from each group read the text with the conclusions drawn from the analysis of data from the class that he worked on and next, there was a moment for debate in which the teacher extended the discussion to the whole class. Sometimes, questions were posed to the entire group. At other times, the teacher asked the questions directly to pupils to extend the discussion and increase participation.

At first, when pupils read the written conclusions, their interventions were mostly about “reading data” available in graphs. They stated the most chosen program, adding the number of pupils associated with that choice, continuing the sequential demonstration of other choices, thus characterizing the television preferences of the class. However, even during the presentation of the findings, having been challenged by the teacher to find "interesting things", they have established comparisons and discovered relationships between the representative values of the various categories. At this stage, the most reported aspects were related with the different choices between boys and girls and the difference between the number of choices of the several programs. Worthy of note is the emergence of the term “balanced” to characterize the similarity of choices of boys and girls or programs with the same number of choices. The intermediate level questions, allowing for comparisons and numerical relationships, contributed to the evolution in the interpretation of the pictograms from "reading data" to "reading between the data."

Having two classes per grade, after the presentation of data from these classes, the teacher challenged pupils to “look closely” at the two pictograms, comparing the results in order to discover “interesting things”. At this stage, he used advanced level questions, trying to get pupils to "read beyond the data" (Curcio, 1989; Friel, Curcio, & Bright, 2001).

This study reveals the teacher’s role in the conduction of discussions, particularly by making questions (Friel, Curcio, & Bright, 2001). João wanted pupils not to be confined to the identification of more and less chosen programs - a characteristic from “reading data” - but tried to get them to evolve into “reading between the data” and “beyond data”. In order for this to be possible, during the discussion he asked intermediate and advanced level questions, seeking to support pupils in identifying certain relationships, or by challenging pupils to find “interesting things”.

The questioning conducted by this teacher, including questions focused on “reading between the data” and “beyond data” helped to promote the pupils’ “graphic competence”, as it is perceived by González, Espinel, and Ainley (2011).

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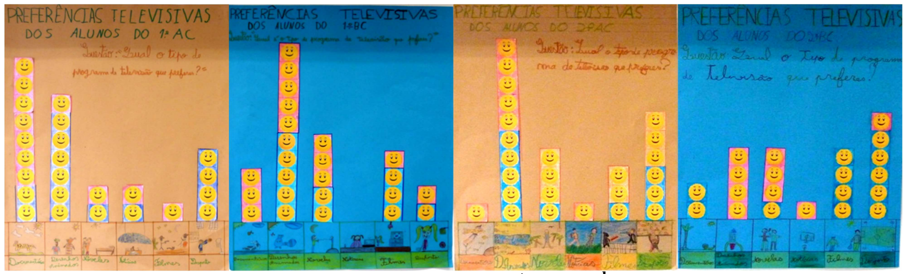
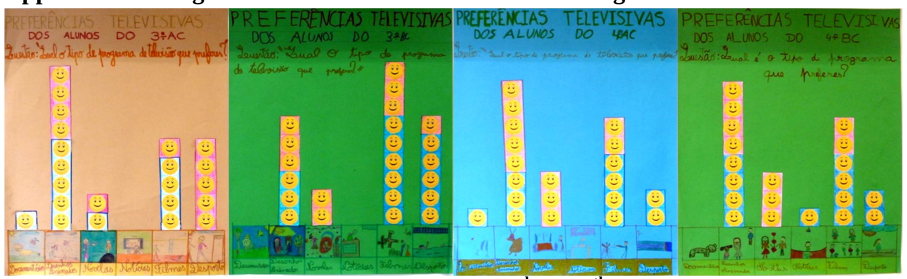
APPENDIX



Appendix 1: Pictograms of the classes from the 1st and 2nd grade.



Appendix 2: Pictograms of the classes from the 3rd and 4th grade.

TEACHERS' PRACTICES AND GRADE 3 STUDENTS' UNDERSTANDING OF BAR GRAPH REPRESENTATIONS

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We aim to understand how Catarina, a grade 3 teacher, promotes her students' learning about bar graphs as statistical representations. The conceptual framework addresses representations and teachers' actions while students are working on tasks. The methodology includes observation and video recording of a class where students solve a task that involves reading and understanding a bar graph. The results show that to promote the understanding of the graph representation, Catarina supported students in reading the data and in reading between the data mostly through guided questioning.

INTRODUCTION

Students' learning is strongly influenced by teachers' practice, particularly by the way teachers use mathematical representations in the classroom (Stylianou, 2010). Representations help students to interpret, organize and understand the information given in the statement of a problem, to formulate solving strategies, and to monitor and evaluate their work (NCTM, 2000). Several authors have been studying and categorizing representations in order to better understand them (Bruner, 1999; Thomas, Mulligan, & Goldin, 2002; Webb, Boswinkel, & Dekker, 2008). At the same time researchers have been interested in how students learn to interpret and use statistical graphs which, as Friel, Curcio, and Bright (2001) indicate, "used for data analysis function as discovery tools at the early stages of data analysis when the student is expected to make sense of data" (p. 132). In this paper, we seek to understand how Catarina, a grade 3 teacher, promotes her students' learning of graph representations.

REPRESENTATIONS AND TEACHERS' PRACTICE

Tripathi (2008) states that a representation is a mental or physical construct that describes the different aspects of a concept, and its connections with other concepts. Duval (2006) and Stylianou (2010) refer the importance of knowing and understanding several representations related to the same concept. Stylianou (2010) also states that the teachers' role is very important in selecting those representations which they consider as more appropriate to promote their students' understanding. In the same turn, Bishop and Goffree (1986) say that the role of teachers is to facilitate the interpretation of representations and to promote the exposure of connections between representations, leading students' to share their knowledge. When students explain their solutions, teachers have the chance to better understand their students' answers and representations. Duval (2006) states that when teachers promote moments of explanation and presentation of students' representations, they are

supporting their students in the establishment of connections between representations, leading them to the make conversions and treatments.

Bar graphs are an important kind of statistical representation. Goldin (2000) indicates that graphs provide opportunities for students to build their own knowledge and to participate in discussions. As Friel, Curcio, and Bright (2001) indicate, all graphs have a resembling framework, but each one has its own specifications and its own language. These authors state that when students read graphs, they must be able to describe, organize, represent, analyze, and interpret data in relation to its context. They regard students' difficulties in reading and understanding graphs associated (i) to the establishment of connections between data, (ii) to the graph with known situations, and (iii) to the task and students' characteristics. To study students' understanding of graph representation, Curcio (1987) designed a framework with three levels: (i) reading the data (direct readings from the graph); (ii) reading between the data (making simple inferences by relating data); and (iii) reading beyond the data (extending, inferring or predicting information based on the data).

Students' learning is strongly influenced by teachers' practices that Saxe (1999) defines as recurrent and socially organized everyday life activities. So, it is important to analyze teachers' practice and how teachers use tasks in a classroom. Ponte (2005) suggests that the class work on a task can be analyzed through three different moments: introduction (which can be made just by the teacher or with a whole class discussion), students' autonomous work (individually or in small groups) and whole class discussion based on students' results. To analyze whole class discussions, Ponte, Mata-Pereira, and Quaresma (2013) propose four types of teachers' actions (inviting, challenging, supporting/guiding and informing/suggesting). Following this perspective, we observed the students' work on a task in order to establish a connection between their activity and the actions of the teacher (Table 1).

Students' activity	Teachers' actions
Designing/Choosing representations	Promoting free choice Hinting through questioning Providing explicit suggestions or examples
Using/Transforming representations	Challenge students through open questioning Asking to explain in a structured way Suggesting alternatives
Reflecting	Guiding or challenging to establish further connections Guiding or challenging to find conversions or treatments
	Promoting the evaluation of the work done Promoting systematizations

Table 1: Teachers' actions regarding students' activity with representations.

Students' activity may involve (a) designing a representation, (b) using and transforming a representation, or (c) reflecting about representations that they used. We seek to relate teachers' actions and students' activity. To support their students in designing a representation, teachers may (i) guide them in a little structured way by promoting the students' free choice, (ii) propose students progressively a more structured path by giving them some hints, or (iii) provide suggestions or give an example that students should follow. To promote the use or transformation of a given representation, the teacher may (i) challenge students through open questioning and lead them to think about transforming their representations, (ii) ask students to explain their solutions in a more structured way, or (iii) suggest explicitly an alternative to the representations that students have chosen. To support reflection, teachers may guide or challenge students (i) to establish connections between the representations (that have or could be used) and (ii) to make conversions and treatments of representations. Finally, the teachers may promote students' (i) evaluation of the work that has been done, and (ii) systematization of information.

RESEARCH METHODOLOGY

This paper discusses an episode from a qualitative research about elementary school teachers' practice involving a group of four Grade 3 teachers that were working in 2013 in a school cluster in the surroundings of Lisbon. The teachers were motivated to promote problem solving with graph representations in their classes. At the beginning of the research (January 2013) teachers were asked to identify some of the topics that they wanted to work with their students. By analysing their students' previous work with pictograms and graphs, the teachers were searching for a task that involved a bar graph because they wanted to address students' needs and difficulties in reading the data and reading between the data. According to that the first author suggested them some tasks. The teachers chose some of the tasks, discussed it in the working group and then made some changes.

In this paper, we focus on one of the teachers, Catarina. We choose her because we consider that some episodes in her class illustrate interesting situations. Catarina is a young teacher with less than 5 years of experience that knew her students from the previous school year, as a mathematics support teacher. According to the teacher, her students had previously done some tasks involving reading the data in bar graphs.

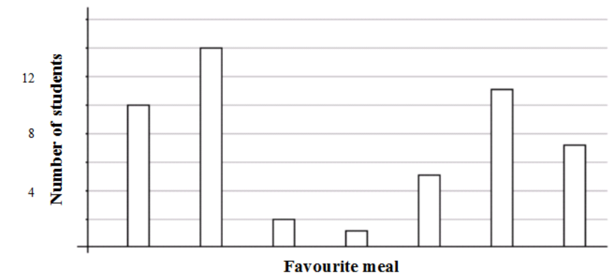
Data collection included video and audio recording during class observation (whole class moments and teacher-individual student interactions). The first author collected the data and was a non-participant observer in the class. Data was analyzed through content analysis regarding the different moments of classroom work on a task (Ponte, 2005), teachers' actions concerning the understanding of representations (Table 1), and the different levels of students' graph understanding (Curcio, 1987). After the data transcription, we analyzed teachers' and students' interactions and we coded their actions using the categories indicated on Table 1 and the levels of students' graph understanding of Curcio (1987). In this paper, we analyze the work carried out on a task (jointly planned by the teachers) on Catarina's class. The teachers working

group decided to propose this task in their classes taking into account their students' needs and difficulties in reading the data and in reading between the data.

RESULTS

The task had two different parts. In the first, the students had to identify each bar graph in relation to a certain meal. To do that, the students should read all seven statements related to the graph representation, which was also part of the statement of the task. The second part of the task had two questions (with sub questions) regarding the data given in the bar graph and in the previous seven statements.

Task – The Favourite meal



The schools' principal wants to know which students' favourite meal is. He received a report that included a graph and several conclusions:

a) The hamburger and fries were the most voted meal;

b) The number of students who chose the hamburger and fries were the double of those who chose roasted chicken;

c) The fish and chips had less 4 votes than hamburger and fries;

d) The spaghetti Bolognese was the second most voted meal;

e) The creamy codfish had 4 more votes than the baked fish;

f) 5 students voted on creamy codfish;

g) Some students voted in peas with eggs.

1) The principal received this little report with the graph but he was not pleased with it. He found the conclusions very confusing. After all, how many students voted? And how many voted for each meal?

2) Examine the graph that was done. What could be improved? What is well done?

Figure 1: Task presented to students (based in Martins & Ponte, 2010).

To interpret the statements a), d), f) and g), the students were required to read the data and to interpret the statements b), c), and e) they should read between the data.

Catarina's class

Before presenting the task, Catarina begins by discussing with her students the graph conventions. As she does that, she tries to understand her students' knowledge about the bar graph representation:

Catarina: (…) Why do we need graphs?... What can we take from them?...

Liliana: Information!...

Flávio: A graph has to have numbers…

Catarina: A graph has to have numbers… Yes… What else?

Joaquim: And it has to have data…

Catarina: Data! In fact numbers… Are data! Aren't they? So let us see… If I wanted to know… In our school… How many students are wearing shoes number 36… What do I have to do?... Which elements should our graph must have? What should we do?

To help students to understand the graph conventions, Catarina questions them in a structured way, giving students some clues ("Why do we need graphs?") and then leads them to understand that graphs have data that can be read, giving them a specific example. After that, she decides to introduce the task by reading the statement of the task and then she questions students about it:

Catarina: …Here we have a graph… What does it tell us?

Jorge: The number of students and their favourite meals…

Catarina: Its favourite meals… So… (she reads the statement of the problem)… What do we have to do? Shall we help the school principal? Here we have some clues, isn't it? Let us look to these statements as clues… And we will discover… Which bar graph corresponds to each meal… Right? Is there anyone who did not get it? (a student raises her arm) You did not understand… So… Statement a) Hamburger and fries was the most voted meal… Right? So we have to take a look to what?... And then we have to try to figure out which bar graph is the hamburger and fries…

While introducing the task, Catarina begins by guiding students in a structured way, suggesting them a path to understand the statement of the problem. Faced with a student that still did not understand the task, she decreases the questioning level and acts in a more structured way, suggesting how to read the data in the bar graph ("The hamburger and fries was the most voted meal . . . We have to look . . . And try to figure out which bar graph is the hamburger and fries…").

During students' autonomous work, Catarina walks around the class and questions the students. She looks into the work of Jorge, a student that apparently solved the task without difficulties and questions him:

Catarina: Jorge… How many students chose… Hamburger and fries?

Jorge: Well [if] here it is twelve (he points to the vertical axis)… Here should be thirteen?

Catarina: Thirteen? Let us see if this is thirteen…

Catarina challenges Jorge to explain his answer through open questioning to assure that he can read the data in the bar graph. By doing that, she notices that the student had answered correctly to the task but he did that without knowing how to read the data (the bar graph scale). Catarina chooses to ask the student to explain her the bar graph scale, guiding him in a more structured way:

Catarina: So [let us look into] the graph… Let us look closer to the number of students… It is jumping how much?

Jorge: …Four and four!

Catarina: …And where is the number zero?

Jorge: Here! (pointing to the right place)

Catarina: So let's complete the scale [in the vertical axis] …So… Four and four… Zero… Four… What about here (she points to the blank space between 0 and 4)? What is this?

Jorge: [The number] two!

Catarina: So write it down… (Jorge completes the graph scale). So… What does it say to us? …How many [votes] are there (she points to the hamburger and fries graph bar)?

Jorge: Fourteen…!!!

Catarina questions Jorge in a structured way focusing him in the graph scale. She also wants to assure that he can read the data (by identifying the bar graph scale and the "0" location). The student seems to finally understand the scale of the bar graph, as he can complete it, and gives a correct final answer ("Fourteen…!!"). After helping Jorge, Catarina continues to help other students and she realizes that some of them are having a hard time in finding the meal that matches to each bar graph. So, she decides to anticipate the whole class discussion, by solving the task with her class, on the board

Catarina: …How many students picked the hamburger and fries meal?

Yara: Fourteen…

Catarina: Fourteen… How do you know that?

Yara: Because… This is twelve… (she points to the blank space that corresponds to twelve in the vertical axis) (silence)

Catarina: And the graph [scale] is jumping how much?

Yara: Two and two…

Like she did with Jorge, Catarina challenges Yara though open questioning to assure that she can read the data. Yara justifies her answer showing that she understood that the graph scale was two and two. Catarina understand her student's reasoning but she feels that a better explanation is necessary (that could enable Yara's colleagues to also understand her reasoning) and she questions Yara in a more structured way ("And the graph [scale] is going from where to where?").

In the last two statements (that is, e) and f)) students showed more difficulties in providing a response as it is necessary to switch the order of the statements. At the beginning of the whole class discussion, António offered himself to participate. Earlier, during the students' autonomous work, Catarina noticed that in statement e) António justified his answer using half of the information provided (if one column is greater than another one and if codfish has more votes than baked fish, codfish will be one column and baked fish another one). So she invites him to present his answer during this discussion of this statement:

António [as he points to the fifth column]: [This is] the codfish…

Catarina: Why?

António: Because it has more than this one [he points to the third column].

Catarina: Hum… How many [votes]?

António: Two…

Catarina: But the codfish has plus four votes than the baked fish… How many votes does this has? [she points to the column chosen by the student]

António: Two…

Catarina: You have two votes…? So two plus four… How much is it?

António: Six...

Catarina: Do you have any [column] with six?...

António: No… Just five…!!!

Catarina questions António in a structured way so he can review the steps of his reasoning. She rereads with the students some parts of the statement of the task, guiding António to read the data and to read between the data. When António justifies his answer, she leads him to read between the data, and afterwards she guides him to focus into reading the data that he had missed earlier (analyzing the value of the remaining columns and comparing it with the scale of the graph).

In the discussion of the second question of the task ("Examine the graph that was done. What could be improved? What is well done?"), Catarina leads her students in how to read the data. Her actions vary widely:

Catarina: …I am looking into… This graph… Tell me something… Is it correct?...

Vanessa: It is incomplete!... It is missing the meals…

Catarina: Where?

Vanessa: Here! (points to the horizontal axis)

Catarina: It is missing the meals… So we could… To complete it, what should we do?

Students: [Write] the [meal] names…

Catarina: [Write] the [meal] names, isn't it?... What else?

Xavier: The title!

Catarina: It's missing the title? Isn't it? If I build a graph like this… It's missing the graph title! A graph must have a title!... Now tell me something… In the [bar] graph can I draw… (she increases the last bar graph width)

Students: Aaaaahhh (reacting negatively)!

Catarina: Why are you reacting like that "Aaaah"?!?!

Leonardo: It is a super meal!!!! (he laughs)

Catarina: A super meal?!?!

Leonardo: Teacher… It is occupying too much space!

Catarina: Too much space… So these [graph] bars…

Leonardo: All the [graph] bars must have the same size…

Catarina: …The graph bars have to be always… With the same width… So… Can they have different sizes? They can't, can they?...

While guiding students in reviewing the conventions of graphs Catarina challenges them through open questioning, so they can find what they can improve ("What's missing?") and asks them in a structured way to explain their answers ("Where? What?"). She also goes beyond what was asked in the task by challenging students and testing them about the width of columns (giving them a "bad example"). So, she challenges students to justify why "all the bars must have the same size" and leads them to show that they understand that in a bar graph every bar has the same width. After the whole class discussion, she systematizes all the information discussed with her students and she writes it on the board.

DISCUSSION AND CONCLUSION

During the class, Catarina's actions are mainly based on challenging through open questioning and questioning or asking to explain in a structured way. Therefore she leads students to explain their answers and to discuss the conventions and rules of graphs, guiding them into the understanding of the main characteristics of a bar graph representation. When students were able to identify and understand the graph conventions it was easier to them to read the data and then to read and establish connections between the data (Curcio, 1987).

Faced with students' difficulties in reading the data or reading between the data, Catarina decreased the questioning level. When she first focused her questioning in reading between the data, she then led students to read the data. She also used different questioning levels (open questioning, questioning in a structured way, and suggesting) according to the difficulties that students were experiencing. On the other hand, when students were having no trouble in reading the data, she increased the questioning level, challenging them so they could read between the data. The decision of increasing or decreasing the questioning level as well as picking the right moment to do it was an important element of Catarina's actions, because she had to be concerned with the risk of jeopardizing the students' activity on the task, making it too easy or too hard.

During the whole class discussion, Catarina chose students with right and wrong answers depending on what she wanted to explore (Bishop & Goffree, 1986).

In conclusion, to help students to read the data Catarina guided her students, questioning them in a structured way. At the same time, she tended to challenge the students through open questioning to help them to read between the data. Faced with students' difficulties in solving the task, she emphasized the whole group discussion, challenging and guiding students to present and to explain their solutions, in order to assure that everyone could understand their colleagues' work. Catarina did mostly challenging actions through open questioning and guiding/supporting actions (Ponte, Mata-Pereira, & Quaresma, 2013), questioning students in a structured way in the three moments of the task, with very little use of suggesting/informing actions.

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SHORT ORAL COMMUNICATIONS

INNOVATIVE DIDACTIC SOLUTIONS FOR STATISTICS EDUCATION IN ITALIAN SCHOOLS

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INTRODUCTION

The Italian National Institute of Statistics (ISTAT) considers the promotion of statistical literacy in young people as a strategic investment for the future of society, since statistics is seen as an essential knowledge to become conscious citizens. Therefore, ISTAT is deeply involved in projects to create innovative didactic products, which may let students learn basic statistical concepts in an entertaining way.

FRAMEWORK

Promoting statistical literacy does not mean teaching technical skills, but rather promoting the ability to critically read and interpret data, and to make decisions using statistics. Statistical literacy is as necessary as other fundamental competencies, such as reading, writing or speaking (Schield, 1999).

Statistics is generally considered boring and difficult by students. Moreover, motivation plays an important role, because learning requires efforts and people do not effectively apply effort without being strongly motivated. Traditional teaching generally does not improve motivation. We have also to take into account that in our communication society young people spend an enormous amount of time consuming digital media. Therefore, the deep interest in technologies young people show could be the key to catch their attention and to involve them effectively increasing motivation. (Prensky, 2006)

As well as technology, learning-by-doing may help increasing motivation and involvement and the comprehension of statistical concepts through directly managing statistical data. It has proved to be one of the most innovative and effective learning method by now, since the use of real data develops critical thinking and the understanding that statistical skills are necessary in everyday life (UNECE, 2012).

METHODS

ISTAT creates didactic products with the help of teachers and students as stakeholders for designing and testing them through focus groups, to devise guaranteed best practices. They can be then standardized and published on the Web in institutional sites for broader use by other Italian schools, accordingly to the open access policy and with support of ISTAT experts. ISTAT may receive feedbacks from users to improve these tools and develop new ones, which is important to create really effective products and thus in bringing statistics even nearer to users.

According to this, ISTAT has recently developed two educational products, focusing particularly on learning-by-doing and technology.

"Facciamo statistica!" (*Let's do statistics!*) is a didactic Kit for students aged 11-13 carried out in partnership with the Foundation Giovanni Agnelli, a private organization in Turin. The focus of the Kit rests in questionnaires on three themes: Environment, Gender stereotypes, Spare time and eating habits. These questionnaires involve increasing difficulty, in order to adapt to students' statistical knowledge level. Students directly experience statistics carrying out statistical surveys in class, both by paper and on line. A methodological manual helps in conducting surveys in class.

"Scuoladistatistica-Lab"" (*Schoolofstatistics-Lab*) is an online platform specifically for students aged 14-18 and it is a *workroom* for statistical knowledge. An interactive and dynamic visualization tool (*Statistics eXplorer*) is embedded. It allows to experience statistics using real data for data analysis and statistical storytelling. It is designed with four growing levels of difficulty to adapt to different levels of students' statistical knowledge and to gradually guide their improvements.

"Facciamo statistica" has been tested by about 700 students of lower secondary schools in Turin and surroundings, while the prototype of the Lab has been tested in upper secondary schools in Rome, Florence, Bari, and Potenza. Testing has been supported by ISTAT experts. Through the feedbacks received, both tools have been suitably integrated/modified and then officially released on ISTAT website.

PRELIMINARY RESULTS

A community of teachers has been created to share experiences using 'Facciamo statistica'. Experiences reported show that the learning-by-doing approach gives better results because it improves motivation and allows students to work cooperatively. The Lab presently counts about 1000 users, who particularly appreciate both the learning-by-doing and the playful approaches. Users, teachers in particular, also appreciate the opportunity to create a community and share statistical analysis.

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EXPLORÍSTICA - ADVENTURES IN STATISTICS

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INTRODUCTION

Explorística - Adventures in Statistics (www.exploristica.com), is an itinerant exhibition consisting of various interactive game-based modules with the aim of bringing the fundamentals of Statistics and Probability to educational communities, conveying the concepts in a practical and experimental way. It is intended for students of upper basic and secondary school (12 to 17 years). Ordinarily, when a student plays a game on a computer, a great deal of data are generated, but never used (Erickson, 2013). In Explorística there are six games and other interactive experiences whose data can be manipulated. It describes five important phases of the statistical process - Select, Collect, Describe, Estimate and Interpret. In the *Welcome Module*, participants introduce their data and produce graphs and tables and discuss the main results. In the *Archery*, players use a real bow to shoot arrows at the target and use distances to the target center in order to learn the concepts of location and dispersion. Inside the *Submarine*, participants perform a journey through an underwater ecosystem where they need to collect a sample of a new species of reptile. Then they produce box and whiskers plots of the data, to help determining the subspecies of the reptile. The *Census Quizz* is a set of questions, based on Portuguese Census 2011 data, which players have to answer in the fashion of "Who wants to be a millionaire". In *False Dice*, participants use four dice, which they roll to discover which of them are fake.

BOX PLOTS AND THE INTERACTIVE SUBMARINE

Boxplots, box plots, or as Tukey box graphs (Tukey, 1977), are graphical methods of representing important characteristics of data, based on the five-number summary of the data: minimum, maximum, 1st quartile, median and 3rd quartile (Everit, 2006). These types of representations, focus on three main features of the data: 1) Center, 2) Spread, and 3) Shape. Few arguments exist against the use of boxplots in the teaching and learning of statistics. However, according to Bakker, Biehler, and Konold (2005), some of the features of boxplots make them particularly difficult for young students to use in authentic contexts. These difficulties include: (i) boxplots generally do not allow perceiving individual cases; (ii) boxplots operate differently than other displays students encounter; (iii) the median is not as intuitive to students as we once suspected; (iv) quartiles divide the data into groups in ways that few students few students (or even teachers) really understand. These authors recommend that educators consider these features as they determine whether, how, and when to introduce boxplots to students. In the Interactive Submarine (see Fig. 1), participants start by seeing a movie in the right part of the module where they are trained about

the goal of the game, and the statistical concepts being learned. Then, they have to capture the reptiles, using the joystick in the left part of the module. A balance, a ruler and a camera are ready to measure weight, age, size in the laboratory at the right part of the module with a 4D (augmented reality) device.

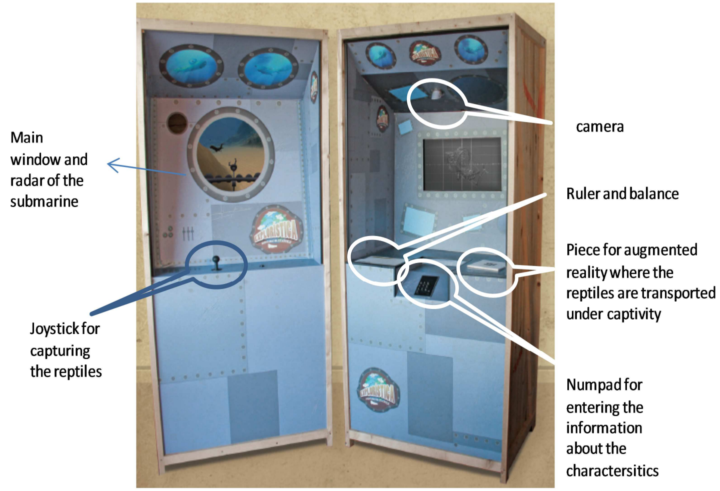

Figure 1: Left part (main submarine window) and right part (laboratory) of the interactive Submarine.

Taking into account the existence of three different subspecies of the reptile in the lake, which are distinguished by their main characteristics (weight, age, and size), participants have to produce boxplots of these variables and compare them with the boxplots of the population to identify the subspecies that have been collected. We observed that younger individuals seem to be more prepared to analyze and interpret boxplots.

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COMMUNICATION, ACTIONS AND QUESTIONS OF ONE TEACHER IN CONDUCTING STATISTICAL TASKS

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INTRODUCTION

The importance of statistics education in the early school years results from its contribution to the development of students' critical reasoning, which is essential to their school life and mainly to their citizenship (Batanero, Godino, & Roa, 2004). This research is part of a broader study aiming to understand the teaching practices of elementary teachers regarding statistics education. In this paper we focus on the types of communication, actions and questions privileged by an elementary teacher in conducting statistical tasks.

TEARCHERS' PRACTICE IN STATISTICS

Several aspects have been recognized as fundamental to the analysis of the concept of practice. Brendefur and Frykholm (2000) highlight the importance of the nature of communication that occurs in the classroom by proposing a model with four types of communication: (i) *uni-directional* communication; (ii) *contributive* communication; (iii) *reflective* communication; and (iv) *instructive* communication. Associated to the type of communication, the type of questions that are being asked are important. Love and Mason (1995) distinguish three types of questions: (i) *focusing*; (ii) *confirmation*; and (iii) *inquiry*. In turn, Ponte, Mata-Pereira, and Quaresma (2013) propose a framework of analysis for teachers' actions in conducting four types of main actions: (i) *inviting*; (ii) *supporting/guiding*; (iii) *informing/suggesting*; and (iv) *challenging*.

Wild and Pfannkuch (1999) consider that statistical work should be developed with students through five basic stages that they call "investigative cycle": (i) *problem*; (ii) *plan*; (iii) *data*; (iv) *analysis*; and (v) *conclusions*. From their perspective, all statistical work focuses on some or all these stages, being possible to analyze teachers' practice, specifically their actions, communications and questions, through the investigative cycle' stages that have been worked in the classroom.

METHOD

The methodology is qualitative and interpretative following a case study design. Data were collected by conducting a semi-structured audio-recorded interview to a 4th grade teacher, and through participant observation of her lessons. Data were analyzed according to the investigative cycle' stages (Wild & Pfannkuch, 1999) and by adapting Brendefur and Frykholm (2000) categorization for the types of

communication, Love and Mason (1995) for the types of questions, and Ponte, Mata-Pereira, and Quaresma (2013) for the types of actions.

RESULTS

Feeling unsatisfied with the previous work developed in class, based on textbook tasks, the teacher decided to propose her students a statistical investigation. In the *problem*, *data* and *conclusions* phases of the investigative cycle, she encouraged students to reflect causing them to engage in the classroom discourse by rising mainly *focusing* and *inquiry* questions, inviting them to participate and guiding their discussions, in some cases challenging them to go further than they did in their autonomous work. In the *plan* phase, the teacher kept the leading role in making the decisions about what to do. In the *analysis* phase, the teacher tended to dominate the discourse, clarifying some concepts and statistical representations, assuming the leading role although putting some *confirmation* questions, which seems to be in accordance with a kind of *uni-directional* and *contributive* communication. This appears to be due to the fact that she perceived the need to address some statistical issues with the class that had already been worked out over the school year, but that this study showed that they had not been understood by the students. Thus, it is possible to observe that although the teacher's types of communication, questions and actions change throughout the work with students, she tended to assume a kind of teaching that highlights students' expression, promotes a *reflective* communication, especially with *focusing* type of questions, and challenges students at specific points, then *guiding* them in the course of their work.

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THE ROLE OF THE CONTEXT DURING A STATISTICAL INVESTIGATION WITH CHILDREN

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INTRODUCTION

During the last years there have been several international recommendations concerning the importance of working with statistical tasks in the early years. According to GAISE report (2005) over the previous 25 years, the learning of statistics has been an important issue in pre-K–12-mathematics curriculum with the introduction of statistical concepts as early as the first grades. NCTM (2000) states that mathematics curricula from prekindergarten through grade 12 reinforces the need to work with students in formulating questions that can be addressed with the collecting, organizing, and displaying the relevant data to answer them.

THE ROLE OF CONTEXT DURING STATISTICAL INVESTIGATIONS

Gil and Ben-Zvi (2011) claim that knowing and understanding the "real-life" context of a statistical task plays an important role in the students' performance. Langrall, Nisbet, and Mooney (2006) stress that the knowledge of context can be an important factor for engage students in statistical tasks. The importance of offering opportunities to students that support them to integrate contextual and statistical information is also discussed in Langrall et al. (2006), who consider that the context knowledge is used by students in different ways: to support data interpretation (most common); critical assessment of the data and in some cases non-productive to the data analysis. Ben-Zvi, Aridor, Makar, and Bakker (2012) discussed the roles and relations of the context knowledge – about the context of the situation of the statistical investigation and data knowledge – arising from data collected during students' informal inferential reasoning. They noted that students develop gradually, but inconsistently, an understanding of making informal inferences using both context and data knowledge.

METHOD

The main goal of this ongoing research is to understand the role of context knowledge during a statistical investigation. This communication presents an experiment of a statistical investigation performed in two lessons with a 2nd grade class. The methodology used qualitative and interpretative approaches and the main instruments of data collection involved observation, audio and video recordings and the documents produced by students. The analysis focused on the presence of context knowledge, during the statistical investigation task, and is based upon the three categories mentioned in Langrall and colleagues (2006) and already presented.

SOME RESULTS AND DISCUSSION

Although the two lessons included in the present study cannot provide conclusive evidence regarding the role of context knowledge, they do illustrate how this knowledge can support students during a statistical investigation. It was observed that, with the use of a real context, the children showed a high involvement with the task. The contextual framework for the initial question of statistical investigation ("Do we weigh more now than last year?") was built using an experiment the class made in the previous year, related to a child obesity project. When discussing the question, it was noticed that the students frequently called upon the knowledge and memories they had from the previous study and of its context, using in their predictions some of the background acquired during the previous experiment. During data collection, some of the children showed some difficulties in understanding the methodology used to collect the data, in some cases due to the context knowledge gathered from their day to day experiences. While in data representation there was no evidence of the use of context knowledge, during data analysis it was noticed that some students used their previously gathered knowledge of the context but in this case some conflicts arose between their context knowledge and the data representation. During the discussion involving the whole class, the context knowledge based on their experiences is used, in particular in association between variables and in finding relationships in the data present in a graph. Finally, this study's main results emphasize the importance of using real context in a statistical task and as it is referred by Langrall and colleagues (2006), the most frequent way the children used context knowledge, in this study, was to support data analysis.

ACKNOWLEDGEMENT

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INQUIRY-BASED TEACHING IN STATISTICS AND CHALLENGES FOR ELEMENTARY AND SECONDARY SCHOOL MATHEMATICS TEACHERS

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INTRODUCTION

Statistics Education remains a challenge for Basic Education in Brazil due to its recent incorporation in the curriculum and the insufficient education (initial and continuing) offered to teachers (Costa & Nacarato, 2011). Such aspects point out the urgent need for materials to support teachers' work and education. Therefore, we have prepared a multimedia case involving Statistics in Elementary School, from the Multimedia Resources in the Education of Mathematics Teachers Project[1], whose aims concern the preparation of multimedia cases and investigation their contribution to the professional development of Math teachers (Cyrino, in press). The purpose of this work is to highlight the challenges presented to teachers in lessons supported on the perspective of the Inquiry-Based Teaching in Statistics (Oliveira, Menezes, & Canavarro, 2013), namely the choice of tasks, planning, communication, meaning and lesson management.

METHODOLOGICAL GUIDELINES

Data for this report was collected during 5 lessons of 50 minutes each, with 32 students from the 9th grade, aged between 13 and 17, from a public Elementary School, in the city of Londrina, Paraná, Brazil. Contents taught in these lessons included averages, arithmetic mean meanings (Batanero, 2000) and some of their properties (Strauss & Bichler, 1988). All lessons were audio and videotaped and, for the analysis presented, these materials were complemented with written productions from students and interviews carried out with the teacher before and after the lessons.

CHALLENGES FOR THE TEACHERS

The lessons analyzed, which gave rise to a case of exemplary practice, show, during the interactions between students and teacher, teacher and the groups and students intra and among groups, some challenges related to the teacher's practice when dealing with Inquiry-Based Teaching in Statistics, which can be summarized as:

- *Task*: task choice is a fundamental element to reach the lesson's objectives, considering, most of all, the importance of prioritizing conceptual statistical aspects over Algebra and Arithmetic. An open and less directive task leads to more promising interactions, reasoning, and registers during the learning process; however, they demand assertive interventions from the teacher.

- *Planning:* to anticipate possible strategies and students' difficulties and misconceptions is fundamental for the development of the lesson and demands consistent statistical and pedagogical knowledge, since it implies questionings, analogies and the retaking of ideas and motivation that promotes the engagement of the students in the task without reducing its level of cognitive demand.
- *Communication*: the teacher plays the role of an instigator/provoker, helping students clarify ideas and strategies, which favors the development of statistical argumentation but also requires the special skill to listen, understand and clarify students' ideas and strategies without directing them.
- *Meaning*: the meaning of statistics concepts depends, substantially, on the context and on the understanding of variability and uncertainty. Thus, to motivate the clarification of registers, algorithms and computations within the analyzed situation is fundamental and requires special skills from the teacher to be able to understand the students' ideas, expand them and relate them to the objectives of the lesson.
- *Lesson Management*: time is a complex aspect of Inquiry-Based Teaching in Statistics since it demands deep knowledge of didactics and experiences for the improvement of the how to manage it during the phases of the lesson.

Such aspects seem to constitute substantial elements to be considered during the educational processes and on the practices of teachers who teach Statistics in Basic Education, to foster the improvement of statistics teaching and learning.

ACKNOWLEDGEMENT

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STATISTICAL KNOWLEDGE FOR TEACHING: THE CASE OF SARA AND MODELLING TASKS WITH TWO-DIMENSIONAL DISTRIBUTIONS

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INTRODUCTION

This communication reports to a wider study about statistical knowledge for teaching. We focus on Sara and our goal is to identify the knowledge that the teacher uses to prepare a modelling task for a two-dimensional distribution, analysing her actions from the conception of the idea for the task to the formulation of the questions.

THEORETICAL FRAMEWORK

There are some inherent difficulties in the teaching of statistics. With regard to two-dimensional distributions, several authors have reported the complexity of teaching and learning data and bivariate relationships (Engel & Sedlmeier, 2011). These relationships often underlie real phenomena models and can be approched by mathematical modeling using statistics – that we call statistical modeling. The modeling is developed through a cycle with several phases, which involved mathematical knowledge and knowledge from other areas, in particular related to the contexts of the reality of the modeled situation (Ferri, 2006). These extra mathematical contexts, were data are immmersed, are recognized as well as necessary knowledge for the teaching of statistics. Groth (2007) proposes a model of statistical knowledge to teach, that we adopt here. Groth advocates the need for common statistical knowledge and specialized statistical knowledge, the first held by those who know statistics and the latest specific to those who teach. Groth also considers that both knowledges may or may not be mathematical (Figure 1).

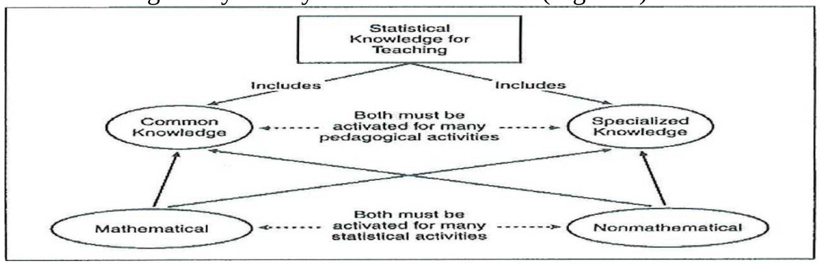

Figure 1: Hypothesized structure of statistical knowledge for teaching.

METHODS

This research is interpretative, following a case study design (Yin, 2003). The data analyzed here refer to a modelling task specifically developed by Sara for her students of professional sports management course. Data collection accompanies Sara since the emergence of the idea for the task to the formulation of the questions.

RESULTS

The idea for the creation of the task emerged from a competition for children organized by Sara students. The teacher promoted data collection and the analysis of the situation by her students, urging them to conjecture about relationships they observed in children. Examples of these conjectures were: "The weight is likely to influence the race ... the heaviest should take longer!" and "The weight can influence the number of hops ... the lighter may jump more often!". Sara took these conjectures as a starting point for the development of the modelling task, in which included photos from the competition, revealing specialized nonmathematical knowledge. She created registration grids for collecting data for her students, showing nonmathematical common knowledge. She included a question asking for "the construction of three types of linear correlation", intending to analyze that the intensity of the linear correlation coefficient can vary. She revealed here some inaccuracies, concluding that her mathematical common knowledge lacks depth. She included in the task issues related to the interpretation of mathematical results obtained from the linear regression line and its confrontation with reality, showing specialized mathematical knowledge. These results highlight the fragility of her mathematical knowledge and the strength of her nonmathematical knowledge – provided by direct contact with reality.

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DEVELOPING STATISTICAL LITERACY IN 5th GRADE: A TEACHING EXPERIENCE

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THE STUDY'S PURPOSE

The purpose of this study is to analyze students' learning in the 5th grade, as part of a teaching experiment aimed to develop statistical literacy, focused on data reading and interpretation. Based on the ability students demonstrate to read and interpret data in graphs, this study sought to answer the following questions: What learning do students show in reading and interpreting bar, line, stem-and-leaf and pie graphs? What are the main difficulties of these students before, during and after the teaching experiment, regarding graphs reading and interpretation?

STATISTICAL LITERACY

Statistics is still often taught as a set of formulas and procedures that students have to memorize and apply in routine problems. However, the increasing need to deal with statistical data in our daily life, led to the rise of the concern regarding statistical literacy. This is the ability an individual has to understand, interpret and critically evaluate data organized in different representations and to appreciate, discuss or communicate their reactions to statistical information, in other words, their opinions about that information and the reasonableness of their conclusions (Ben-Zvi & Garfield, 2004). Students' ability to read graphs has received increasing importance in research. The graph sense or understanding involves reading and making sense of it in real life situations, as well as the ability to create graphs that best convey information and data (Bright, Curcio, & Friel, 2001). Curcio (1987) defined three levels of graphs reading and understanding: Level 1: *Read the data* - Direct reading of a graph, without any interpretation, given the facts only explicitly represented; Level 2: *Read between the data* - Requires some comparison, with knowledge of mathematical concepts and skills; and Level 3: *Read beyond the data* - Requires an extension of the concepts, prediction, or inference.

RESEARCH METHOD

This study follows an interpretative and qualitative research methodology. A teaching experiment was developed with a 5th grade class who solved ten research/exploration/problem tasks, during seven 90 minute lessons. The author assumed the dual role of teacher and researcher. The students have previously worked with graphs in the 3rd and 4th grades. Different methods were used in data collection: participant observation; documentation collection; semi-structured interview (three case studies); initial and final tests. The categories considered were: the strategies

used by students in the exploration of the proposed tasks; levels of reading/ comprehension and interpretation of graphs; the difficulties they manifested.

CONCLUSIONS

While solving the proposed tasks, with regard to issues involving literal reading of graphs most students did not reveal difficulties. As Curcio (1987) contends, this is due to the fact that the students do not have to perform an interpretation of the data, resuming themselves to only extract the explicit information in the graph, or in its elements. They also show difficulties in handling rational numbers in pie charts especially concerning the notion of percentage. Regarding level 2 and 3 questions, students reveal more difficulties, especially when they need to apply other mathematical knowledge to answer the questions. Throughout the teaching unit, students reveal difficulties in written communication. It is also to be noted that they had great difficulty in summarizing graphs as they tend to merely enumerate what is explicit in there.

In the course of teaching unit, students evolve positively being able to better explain their reasoning and be concentrated and attentive to every detail of the graphs. By the end of teaching unit, they are able to make a literal reading and extract explicit data in charts and also interpret them. The students reveal progress in their mathematical communication skills, with increasingly clear and elaborated answers, presenting their reasoning in an understandable way. They are already able to compare representations or use other mathematical concepts, such as the case of the four basic operations or apply their previous knowledge of rational numbers as a way to help them to read and interpret pie charts, thus starting to develop their statistical literacy.

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5th GRADE STUDENTS' INFORMAL INFERENTIAL REASONING WHEN USING TINKERPLOTS

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INFORMAL INFERENTIAL REASONING IN THE EARLY GRADES

Statistical inference is both a central objective of statistical reasoning and one of the most difficult ideas for students to apprehend. Recognizing the fact that students have some sound intuitions about data, recent curriculum orientations and developments in statistics education suggest the promotion of students' inferential reasoning from early years of schooling (e.g., Ben-Zvi, Aridor, Makar, & Bakker, 2012; Franklin et al., 2005). This exploratory study documents the effort to facilitate the emergence of informal inferential practices with 5th grade students when exploring tasks oriented for statistical reasoning supported by the *TinkerPlots* for data handling. In this communication we focus, particularly, on what aspects of informal statistical inference (ISI) emerged in students' work and on their difficulties regarding those practices.

Informal inferential reasoning (IIR) is described as an informal process of "making probabilistic generalizations from (evidenced with) data that extend beyond the data collected" (Makar & Rubin, 2009, p. 83) involving: generalizations that extend beyond the data; the use of data as evidence for generalizations; and the use of probabilistic language in describing the generalizations including reference to levels of uncertainty about the conclusions drawn. In an attempt to articulate the everyday data-based classroom practices and the possibilities they present to develop students' IIR, Leavy (2011) suggests certain activities such as those that: (i) require students to look at the sample data first, in an effort to identify underlying patterns and then to make generalizations beyond a sample to the population; and (ii) involve the comparison of two data samples to ascertain the existence of differences between populations. Moreover, task's selection to support IIR can be informed by the degree to which they require students to get involved in the above described basic components of IIR (Makar & Rubin, 2009). Several studies on the affordances provided by dynamic statistics learning environments, such as *TinkerPlots*, for making inferential reasoning accessible to young students (Ben-Zvi et al., 2012) also found very encouraging results.

METHODOLOGY

This study arose in the context of a developmental research project (DRP) which strives to develop understanding of how sequences of instructional activities, based on *TinkerPlots'* use, promote students' statistical reasoning. It involved 19 students

considered quite problematic by the teacher (the first author) who participated in the above referred DRP. Data collection was carried out during four 90 minute lessons, and included: (i) the students' written work on two tasks that present the features suggested by Leavy (2010) and enable them to experiment meaningful aspects of the statistical inference practice in an informal way; and (ii) screen records of their work. A qualitative and interpretative data analysis, using the framework by Makar and Rubin (2009), provided insights on characteristics of ISI in the students' work.

RESULTS AND DISCUSSION

When solving the tasks the students formulated questions and conjectures based on available data. However, their work did not show clear characteristics of generalization based on data or the use of data as evidence for the generalization. In fact, students made inferences based on their everyday knowledge, as they presented statements supported in their experience and/or personal beliefs, not recognizing the need for data to make decisions and evaluate information. Moreover, students' conclusions reflect that they are able to analyze representations they did with *TinkerPlots* but they simply make a direct reading of the data and do not understand that those vary, sometimes predictably, being potentially useful to explain or predict trends. Thus, students' inference making was still limited. Students also used probabilistic terms in some of their conclusions, though without reference to levels of uncertainty. While the interactions between students during their work on the tasks had an important role in familiarizing them with the software and some notion of informal inferences emerged, the identified difficulties reinforce the need of working for an extended period on these ideas from the early years of schooling.

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STATISTICAL LITERACY IN TEACHING OF NATURAL SCIENCE

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TYPES OF STATISTICAL LITERACY

It is debatable if teachers of natural science should be statistically literate. There are several reasons for statistical literate teachers:

Understanding common situations is important because media use numbers and statistics on a daily basis (Watson, 1997). But there are lot of people how don´t understand them. Teachers as people transferring knowledge should understand any newly published research or statements in their field.

Students have to **write their final thesis** which usually has a practical part as well. Such part contains statistical evaluation of data. However, students who never had any subjects related to statistics or quantitative research can experience problems in using statistical methods when they need to evaluate data.

Sometimes it is much better to **provide pupils with some real-life example** than to merely explain the subject matter. Ideally through, an interdisciplinary connection should be employed. Nature science subjects can be easily connected with mathematics. For example, the teacher should collect data from student experiments and finally make some comparison of results.

The following reasons imply three types of statistical literacy (Fig. 1).

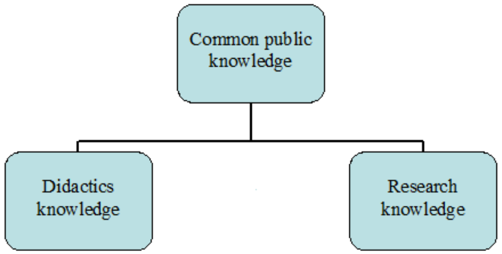

Figure 1: The division of statistical literacy into different types of knowledge.

Common public knowledge

Each person should understand the socio-political situation of his or her country. Information about policy, economy and social affairs are presented in newspapers, televisions and internet every day usually via numbers (mean of wages, percentage of preferences or estimated inflation rate). Comprehending the information should be learnt at elementary and high schools. That covers only descriptive statistics, such as mean, mode and some basic information about variance and probability.

Didactics knowledge

Teachers of nature science conduct experiments to explain students the validity of global principles (in the sense of Burgess, 2011). During the evaluation of the examples the teacher gains data which needs to be processed, analysed and interpreted. The process should be demonstrated to students because it is only practical usage of mathematic in interdisciplinarity with natural sciences. The connection between research methods and teaching practice is described by Garfield & Ben-Zvi (2008).

Research knowledge

A modern teacher should also have the ability to study the latest trends in his or her field. Every teacher comes across statistical terminology in books and articles. That is very often the case in natural science and in research in pedagogy. It is necessary to educate student as future scientists and encourage them to use scientific methods (Rumsey, 2002). Students should be able to ask questions, collect data, use statistical tools, interpret results and draw conclusions. Students are also going to write their final thesis. Tulder (2012) defines types of research for finishing bachelors and masters. Bachelors make a research based on their own questions. A broad spectrum of research methods is understood so that a student can choose the best method concerning the research topic. Finishing master students translate other people's experience into their own research questions. They should know a wide range of research methods (Tulder, 2012). Research knowledge also has to be provided by universities because a diploma thesis is a proof not only of the quality of the student but it also shows the quality of the university.

HIGHLIGHT

All three types of described types of statistical literacy should be provided by universities. The author recommends implementing some interdisciplinary overlaps to subjects which will ensure complexity of teachers' education. However, this is not current practice in the Czech Republic.

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RELATIONSHIPS BETWEEN HYPOTHESIS TESTING AND PROBABILITY/OTHER INFERENCE TOPICS

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RESEARCH BACKGROUND

Garfield and her colleagues (2005) emphasized the importance of statistical literacy and reasoning in college level statistics education; statistical literacy refers to "understanding the basic language of statistics" (p. 14), and statistical reasoning refers to understanding statistical processes and interpreting statistical results. However, university students in introductory statistics courses commonly hold misconceptions about the terms used in inferential statistics (Castro Sotos, Vanhoof, Can den Noortgate, & Onghena, 2007). For example, Vallecillos and Batanero (1997) found that college students commonly develop misconceptions in their understanding of hypothesis testing at the introductory statistics level. According to Castro Sotos et al. (p. 103), "[t]he main reason for this phenomenon is that performing these [hypothesis] tests requires students to understand and be able to relate many abstract concepts such as the concept of a sampling distribution, the significance level, null and alternative hypotheses, the *p*-value, and so on." Taking up this line of research, the study reported in this paper inquires into how closely students' understanding and performance of hypothesis testing (HT) are related to their understanding of the topics of 'null and alternative hypotheses' (NA), 'sampling distribution' (SD), 'rejection region' (RR), 'confidence interval' (CI), '*p*-value' (PV), 'conditional probability' (CP), 'significance level' (SL), 'type 1 and 2 errors' (ER), and 'Modus Tollens reasoning structure' (MT).

To assess student understanding of these ten topics, the study uses three parameters: lexical understanding (knowledge of definition), procedural performance (computational ability) and conceptual understanding (understanding statistical concepts and developing interpretation skills). Lexical understanding is assessed because while the development of statistical literacy requires a basic understanding of statistical terminology (Rumsey, 2002), the literature shows that students develop misconceptions of statistics terms. The assessment includes students' procedural performance because while calculations do not demonstrate understanding of statistical ideas (Garfield et al., 2005; Rumsey, 2002), calculation is still integral in the textbooks currently used in the US. Further, assessing conceptual understanding is critical because statistical reasoning, as Garfield et al. (2005) assert, implies understanding statistical processes and interpreting statistical results; students can improve their statistical competence by understanding statistical concepts and interpreting results (Rumsey, 2002).

RESEARCH QUESTION AND METHODS

The main purpose of the study is to show how student learning of HT relates to the learning of other statistics topics. Using the abbreviations for the topics, I ask the following research question: To what extent does students' lexical understanding, procedural performance and conceptual understanding of the nine topics – NA, SD, RR, CI, PV, CP, SL, ER and MT – relate to their learning of HT? This study further aims to understand the extent to which students' lexical understanding, procedural performance and conceptual understanding of the nine topics relate to one another.

The study's participants are 242 students enrolled in elementary statistics courses in a university in the US. The data are being collected via three surveys, each of which has 10 to 13 questions. The design of the survey questionnaires was inspired by the database from the ARTIST, the items used by Castro Sotos, Vanhoof, Can den Noortgate, and Onghena (2009) and my own experience. The questionnaires are intended to assess the three aspects of student learning (lexical understanding, procedural performance and conceptual understanding) on the ten topics.

ANTICIPATED RESULTS

I anticipate that the study results will show which topics have strong correlations with HT in the three aspects of student learning of HT. The results will shed light on which topics statistics instructors and educators need to focus on to improve students' learning of HT in college level elementary statistics courses.

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SIXTH-GRADERS INTERPRETATION OF INFORMATION USING BAR GRAPHS AND ISOLATED CASES

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This study aims to understand the effect of different representation of information using clustered and stacked bar graph, and isolated cases on Portuguese children's mathematical reasoning, when discrete variables are involved.

Nowadays, citizens have to deal with data information presented in several ways, specially tables and graphs, demanding the ability to analyse and interpret information. Statistic is one of the most relevant domains of the citizen daily life (Steen, 2002). The role of statistics on the understanding of the social reality and also its application on other domains makes it relevant since elementary school levels. Beyond this, Statistic may promote the development of students' critical sense, which is fundamental for students in particular, but also for citizens in general (Batanero, Godino, & Roa, 2004). Statistic is one of the most relevant domains of the citizen daily life. It may promote the development of students' critical sense, which is fundamental for students in particular, but also for citizens in general (Batanero, Godino, & Roa, 2004). Students should be aware of the power of the different types of representation of information and its adequacy according to the situations and nature of data. Bar graphs provide representation of information that students should be able to build, interpret and use in solving problems and investigations. This study focuses on Clustered and Stacked bar graphs that can be used to describe two or more categories of a variable. Visually, Clustered bar graph provides first the comparison of different categories and after the comparison of those of the axis of categories. In Stacked bar graphs each bar is divided into the number of categories, using different colours or patterns. One of the problems of this representation is that the frequency of the first category can compromise the reading of remaining categories. Watson and Moritz (2001) argue that students present different levels of response to the representation and interpretation of information, and pictographs can be seen as a starting point for the development of the analyses of relations between variables. Also Selva, Falcão, and Nunes (2005), who analysed the combination of isolated cases and bar graphs on 39 children (6- to 8-years-old) about the understanding of additive concepts, argued that isolated cases can have an important role on children's understanding of bar graphs. Concerning the representation of discrete variables, Nunes (2004) argues that the use of isolated cases emphasizes the direct connections between the signs and the units they refer to, making them similar representations of the information; tables and graphs belong to the class of symbolic representations as they refer to relations between variables. Carvalho (2008) investigated the influence of staked bar graphs, tables and isolated cases on 12- to 13-years-old children's (n=226) understanding of multiplicative concepts. Results suggest that tables and graphs enhance children's performance in tasks oriented to assess whether there is an association between two variables.

This study addresses three questions: 1) Are there differences on children's performance when the information is represented by clustered bar graphs, stacked bar graphs and isolated cases?; 2) Are there differences on children's reasoning in each of these conditions?; and 3) What difficulties do children present when solving problems with information presented using these representations?

A survey by questionnaire was conducted with 6th graders (n=120) from public Elementary Schools, from Braga and Porto. The tasks are adapted from the study of Carvalho (2008). The children were randomly assigned to work in one of the three groups: *Clustered Bar Graph* - solved the problems using only clustered bar graph; *Stacked Bar Graph* - solved the problems presented only using stacked bar graph; and *Isolated Cases* - solved the problems with the information presented by isolated cases. Each group solved 5 problems to interpret information; the problems were controlled for across the groups. Data collection is completed and analysis is still being carried out.

Preliminary results have been suggesting differences on students' performance and understanding of information according to the type of representation. Educational implications of these findings will be discussed.

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DON’T PANIC: IN-SERVICE TEACHERS’ ATTITUDES TOWARDS STATISTICS FROM ELEMENTARY SCHOOL

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INTRODUCTION

Teachers’ attitudes towards statistics can have a significant effect on their own statistical training, their teaching of statistics, and the future attitudes of their students. In Portugal, research of attitudes towards Statistics has a first Ph.D. work published about Portuguese teachers’ attitudes towards Statistics (Martins, 2015). This work is part of a broader study and focuses on the measurement and description of attitudes towards statistics of Portuguese in-service teachers from the 1st cycle of basic education (ages 6 to 9). The instrument used is the Scale of Attitudes Towards Statistics (EAEE) by Estrada (2002). With this work we hope that an attitudes’ pedagogy may be introduced. The main aim here is to summarize this first assessment and describe attitudes towards statistics of Portuguese teachers’ from the 1st cycle (pupils aged from 6 to 9). The specific objectives are: studying these teachers’ attitudes as a global measure and using their components (Estrada, 2002; Martins, 2015); analysing the relationships between teachers’ attitudes with demographics and school training variables. Some of this information is also crossed with the one of our (qualitative) study by Martins, Nascimento, and Estrada (2012).

METHODOLOGY AND RESULTS

The research method was based in the broader study developed that was a mixed study (Martins, 2015). The instrument used was the EAEE (Estrada, 2002; Martins, Nascimento, & Estrada, 2012), a scale specially design for teachers. There are three generally accepted pedagogical components of the term “attitude” (quoting Aiken, 1970; Auzmendi, 1992; Gómez-Chacón, 2000; Olson & Zanna, 1993 in Martins et al., 2012): (a) Affective: feelings about the object in question; (b) Cognitive: the person’s self-perception as regards the object; (c) Behavioural: the person’s inclination to act towards the attitude object in a particular way. Estrada (2002) complemented these threefold classical attitude components with three anthropological components: (d) Social: perception of the value of statistics in society; (e) Educational: interest in learning and teaching statistics; (f) Instrumental: perceptions of the use of statistics in other areas. Three Portuguese regions were surveyed with EAEE in a total 852 in-service 1st cycle teachers’ valid questionnaires. Teachers’ explanations of nine items were studied using content analysis (Martins et al., 2012). This study confirmed the high internal consistency of the instrument with a

Cronbach alpha of 0,878. The multidimensional aspects of the EAEE – Estrada’s components - also emerged. The comparison with EAEE in other countries reinforced the acceptability of these results (Martins, 2015). Teachers in this study presented a positive attitude towards statistics, since their mean global scores and standard deviation are: 87.97±11.87 (globally varying from 25 to 125, and mean point scoring 75). This study highlighted in a positive way the cognitive and social components and in a less positive way, lower scores, the behavioural and instrumental components (cf. Martins, 2015, pp. 261-288). The attitudes towards statistics of these teachers were not significantly related with their gender but were significantly related to their teaching experience (younger teacher with a median score greater than older ones), their training in statistics (teachers with specific training in statistics in their degrees had better mean scores) and the teaching of statistics (teachers that had already taught statistics had better mean scores). In Martins and colleagues (2012) we analysed some of the available open-ended questions to know the reasons of the in-service teachers to their Likert scale options, and in that analysis we could perceive that they saw that “statistics is not only valid for scientists”. Those in-service teachers also revealed that they generally like learning and teaching statistics, and they see it as a tool to face real-world problems objectively (Martins et al., 2012).

CONCLUSION

In-service teachers have a clear conception that statistics is useful and they value its role in the citizens’ daily life. They also have a clear sense of the importance of including it in current curricula. Although contradictory, outside the school teachers do not see statistics as a tool in their own daily life and they also express a feeling of disbelieve towards its use and the information in television. Even with general positive attitudes towards statistics, from the perceived contradictions it is necessary to clarify them, maybe now analysing in-service teachers’ attitudes towards statistics individually interviewing them and also analysing their classroom practices.

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REASONS TO CHOOSE A STATISTICAL GRAPH: A STUDY WITH TEACHERS OF ELEMENTARY SCHOOL

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INTRODUCTION

In this article we seek to find answers to the questions: a) What types of graphs are usually chosen by teachers in the early years of school in the teaching of Statistics?; b) What are the reasons that lead teachers to choose these graphs?; and c) What are the strategies and contents of teachers' feedback to students when choosing a particular graph? To answer these questions we conducted an open-ended questionnaire based on Ferreira (2012) work and after, we did a preliminary study about the use of statistical graphs by teachers in the early years of elementary school in Brazil. Ferreira (2012) points out that primary teachers face different concerns comparing to secondary teachers when dealing with graphics in the classroom. For example: 1) The degree of scientific rigor in the construction of graphical representations; 2) The better graphical representation for the statistical study; and 3) correct interpretation of the data by the students. Consequently, teachers' decisions have an impact in the learning of students. In the teaching of Statistics, the previous knowledge that a subject holds about a chart is indicative of the fact of having been (or not) exposed to a previous situation with the same type of representation (Curcio, 1987).

Alacaci, Lewis, O'Brien, and Jiang (2011) have also conducted an investigation with teachers in their initial training on their ability to choose the more appropriate graphics to a given set of data. They point out that different explanations support teachers' choices: the type of task in question, the structural components of the graphics and even reviews and/or personal preferences

METHODOLOGY

The approach to this study was based on a qualitative methodology of data collection and analysis. First a questionnaire was applied to 31 teachers from the metropolitan region of Recife. Ages of the participants ranged from 26 to 53 and the length of service was 7.5 years, on average. The questionnaire consisted of 27 items, 11 about the characterization of the teacher and of his/her performance in class when teaching Statistics, 12 items on the selection of graphs and 4 questions on the feedback of the teacher when using graphics. In the questionnaire, we selected graphics depending on their semiotic complexity (Arteaga, 2011). Similarly, we considered the math curriculum guidelines in different Brazilian documents for elementary school teaching. In open-ended questions, teachers reveal why they choose certain graphs and their responses were analyzed using a content analysis.

RESULTS

Regardless of the level of performance of their students in Statistics, 74.1% of teachers place the bar graph in the first position of their preference list, and that also includes line graph, scatter graph and pictorial graph. However, teachers of students with higher performances in Statistics put the pictorial graph as their first choice in the list for the more suitable graphics to work with in class. The scatter plot was selected by 77.4% of teachers as the less appropriate, independently of the performance of their students and independently of the degree of semiotic complexity, the bar graph remained as the type most reported. In this, we identified six categories of explanations on why teachers place the bar graph as the first: 1) It is more easily understood by students; 2) It is the most frequent type of graph found in textbooks; 3) It is more familiar and appropriate for students' school level; 4) It promotes students' attention and engagement; 5) The structural components of the graph, 6) It is makes teaching easier. As for other types of graphs, we summarised participants' explanations into four categories when selecting those: 1) The structural components of the graph; 2) The ease or familiarity in teaching; 3) The ease in understanding and interpreting the graph; 4) Personal preferences. Some of these categories are according with Alacaci et al. (2011) study. Our results also suggested that teachers use a diversity of strategies and contents of feedbacks when working with statistical graphs with their students. These strategies and contents related essentially with collaborative work, the frequency of classroom activities based on graphics and finally the resort to interactive materials.

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LEARNING STATISTICS THROUGH R PROGRAMMING: AN ACTIVITY FOR LINEAR REGRESSION

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Environmental scientists rely on probability and statistical knowledge for experimental data analysis, decision-making, supporting theory, and communication of scientific results. However, many efforts to include these topics in biological sciences graduate programs, have had unsuccessful results (Bishop & Talbot, 2001). In our project [1], begun 4 years ago, we have been researching how to enhance the teaching and learning of experimental analysis and statistics for university under- and post-graduate environmental and biological sciences students, using the constructionist philosophy (Papert & Harel, 1991) –which suggests that learning can be facilitated if students engage in exploring ideas and concepts through construction. Thus, using an iterative-design methodology, we have been developing sequences of constructionist and collaborative activities where students engage in computational programming tasks (using R code –http://www.r-project.org/) through which they can explore, implement and develop meaning for statistical concepts. These activities have, so far, been implemented in 12 university courses in three institutions in Mexico and Portugal, where preliminary results (see Mascaró, Sacristán, & Rufino, 2014) from qualitative, and some quantitative, data of students' work, show marked improvement in students' appreciation for statistics; some appropriation of the concepts and of the technological tool (R); and a mild improvement in regular school assesment scores. We are currently working on developing better assesment and research methods.

As an example of our work, we present the structure of a programming activity to explore the concept of linear regression. It consists of a simulated data set with three vectors in columns identified as either explanatory (*x*) or response variables (*y1*, *y2*). Instructions and tasks are given directly in R-script. In Task 1, students retrieve data from an Excel file using commands they are familiar with, and explore its structure; the objective is to recognise that items in rows follow a one-to-one (*y*-*x*) relationship, and *x* is a continuous (as opposed to categorical) variable. Then, in Tasks 2-3 students construct *y*-*x* graphs (Fig. 1) and answer questions about the size and numerical measurement of variation in *y1* and *y2*, and its relation to *x*; the objective is to compare situations where the response depends, or not, on the explanatory variable, and to associate this to relative measures of variation observed in the graphs. Students then adjust linear models to the data sets and examine the ANOVA table (Tasks 4-6). They are requested to identify those values in tables that represent (residual and total) *y*-variation due to changes in *x*; explain how *F* and *p* values are obtained; and

interpret the result of each test in terms of the coefficient of determination. This helps them relate the graphical and numerical account of variation in the response, to its attributable sources (*x*, residual), the statistical connotation of the result, and its representation within the *F* distribution for a given set of degrees of freedom. Then, in Task 7, by adding two straight lines to each *y*-*x* graph (Fig. 1) –representing, correspondingly, the mean *y* value (blue), and the (red) regression line derived from the adjusted model in each case–, students compare both lines with the dispersion of the data, visualize how the regression line is adjusted by predicting the most probable *y*-value for each *x*, and see how this procedure generates different expected lines depending on the nature of the dispersion patterns in the data. In Tasks 8-9, they obtain and identify regression coefficients, estimates of the residual variation, and use the information to estimate a 95% CI for both slopes and intercepts, which helps them to learn to interpret output elements that result from inputting commands, and relate them to previous graphs and numbers. The benefit of these tasks is that students are not following instructions or giving commands blindly; rather, by actively engaging in coding (e.g. for building representations) they need to understand at each step what they are doing, thus building meanings for statistical commands and concepts.

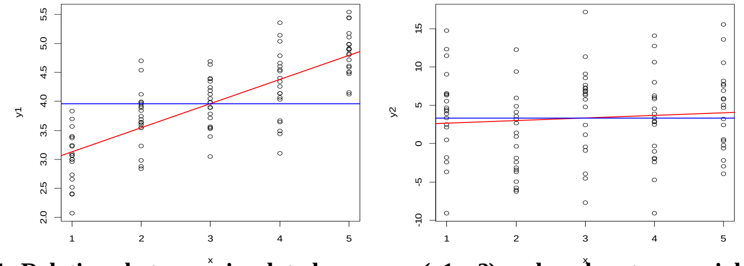

Figure 1: Relations between simulated response (*y1*, *y2*) and explanatory variables (x) and lines generated by the students answering to instructions 2, 3 and 7.

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GRAPH READING AND INTERPRETATION: AN INVESTIGATION OF INTRODUCTORY STATISTICS COLLEGE STUDENTS' KNOWLEDGE

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RESEARCH BACKGROUND

Data visualization, the use of graphs and images to represent data or information, is beneficial to any sort of business. The ability to analyze, interpret, and communicate information from data are considered essential for our daily life (Perez & Febles, 2006). Even though data visualization has tremendous potentials a general lack of understanding is observed. Statistics education is becoming the focus of reformers in mathematics education as a vital aspect of the education of citizens in democratic societies (NCTM, 2000). But still, recent work in statistics education reveals that students are likely to have beliefs about the features of graphs that are different from what is expected (e.g., Meletiou & Lee, 2002). It is generally accepted that the competence related to statistical graphic language is not achieved in the students (Arteaga, Batanero, Contreras, & Cañadas, 2012).

When we refer to graph reading we actually refer to obtaining data directly from one or more graphs and generating information by using explicit data shown in one or more graphs. This requires the recognition of graphical conventions and the making of connections between context and data (Curcio, 1987). Graph interpretation is used to acquire an opinion about a graph; it is used to compare different set of data displayed graphically; it is used to identify any patterns and trends; it is used for making inferences from graphs. Nevertheless, an effective interpretation is possible only if the reader possess basic graph reading skills. Thus, readers' interpretations of a graph provide, at the same time, evidence of their knowledge of the graph's structure (Friel, Curcio, & Bright, 2001). We want to measure the level that such knowledge reaches (regarding graphical understanding) as these are described in Arteaga and colleagues (2012).

RESEARCH QUESTIONS AND METHODS

The purpose of this study is dual. First, we aim at examining the "readiness" of undergraduate students of introductory statistics course into reading and interpreting graphs. Secondly, we wish to investigate how the use of new technological tools, specifically TinkerPlots, affects students' perceptions of graphical representation of data. We are interested into observing the evolution of their pre-knowledge and how their graphical understanding changes when using TinkerPlots.

The site for the study is an introductory 1-st year statistics course at a university in Cyprus. There are 72 students in the class most of whom were majoring in education

sciences or a business related field of study. The data are collected using two pre-tests and in-class assignments. The tasks that formed the pre-test were selected from the Item Database of the Assessment Resource Tools for Improving Statistical Thinking website (ARTIST). Additionally, students were asked to present (in groups) a short survey on a topic of their interest. This forced the students to use TinkerPlots for graphically representing their data. A post-test was also given to measure their progress.

ANTICIPATED RESULTS

What we anticipate from this study is to locate specific students' difficulties with graphs (we expect students to have a very poor understanding of graphical representations) and moreover determine up to which level the use of technology helps/or not their graphical understanding in the critical reading of data. These results will help us educators to concentrate on the Cypriot students' needs and further reform statistics education curricula in Cyprus.

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COLLABORATIVE WORK AS A TOOL TO PROMOTE STATISTICAL LITERACY IN AN ELEMENTARY SCHOOL

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INTRODUCTION

This paper refers to a research developed by a collaborative group of teachers and researchers which explored an interdisciplinary teaching approach related to statistical literacy in elementary schools. Fernandes, Carvalho, and Correia (2011) investigated the characterization of teaching statistics in Portuguese schools. They identified that, despite the teachers recognize the statistics importance for the people’s daily life, they emphasize a teaching approach that reduces statistics to standardized procedures for data handling. The results suggested that the teachers had difficulties to address the complexity of statistical literacy, which is not comprised only of technical components, but of subjective ones such as beliefs and attitudes towards the data (Gal, 2002; Queiroz, et al. 2015). Studies based on qualitative methodology and with the teachers’ participation in the research process suggest that these teachers engage in an active professional development in which they explore new knowledge and opportunities for what is taught at school (Fiorentini, 2013). Although this participative process seems to be constructive, some teachers’ beliefs and attitudes on statistics teaching may persist (Souza, Lopes, & Mendonça, 2014).

METHODOLOGY

The research was qualitative in nature and it had a collaborative approach developed at a Portuguese elementary school. The group was formed by five math teachers and three researchers. All the seven meetings were performed at school. During three meetings the group discussed aspects of statistics teaching to promote statistical literacy, which include the development of a critical stance while interpreting statistical data. The group planned an intervention of two meetings in which two class groups were jointed. The teaching situations were held within the curriculum area called CSI (Integrated Social Competences) that address transdisciplinary contents. The planning was a result of suggestions brought by all members of the collaborative group. These suggestions were analysed and criticized by the group and after the implementation of tasks during the teaching situations, the whole group reflected about the actions. The activities and tasks involved problem situations in which were used different resources and tried to link everyday situations with the statistical content. The research data collection was based on field notes taken after the collaborative group meetings by the three researchers and during the pedagogical interventions about analyses of statistical data with the students. These notes were

shared with all members of collaborative group which added comments and information. A content analysis of the notes was developed associated with theoretical reflections from other studies that also discuss collaborative approaches (e.g., Fiorentini, 2013).

RESULTS

The results indicate that from the processes of discussion, planning, implementation and evaluation of the situations related to statistics teaching emerged challenging issues to the participants (e.g., different ways to explore the investigative cycle in the teaching of statistics). For instance, the teachers' expectations regarding the students' understanding and engagement influenced the organization of tasks in such a way that they have been simplified. The meetings arranged under the CSI were an interesting aspect, because the students could participate in different ways than in more formal disciplines, such as mathematics. For example, the fact that the statistics content was associated with videos and stories about the day to day allowed less formal interactive situations and greater involvement of most students. The discussions which emerged from this project also offer indications for effective in-service teacher education strategies based on teachers' collaborative and reflective participation. Therefore, for example, the experience of sharing ideas and difficulties about how to teach elements of statistical literacy enabled the group to reflect about the possibilities to use outside school resources, such as media graphs and adverts. Future research with similar approaches could investigate aspects of attitudes and beliefs of the participants, since these elements were important in situations that aimed the development of statistical literacy.

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TEACHERS' PERSPECTIVES ON PROMOTING A STATISTICAL REASONING LEARNING ENVIRONMENT

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BACKGROUND

This study comes from a developmental research project (DRP) that sought to promote students' statistical reasoning, in general, and their informal inferential reasoning, in particular, as they solved a sequence of tasks supported by the use of *TinkerPlots* software. A group of middle school teachers and researchers were involved in the process of construction and class implementation of these tasks, which were supported by the principles of the Statistical Reasoning Learning Environment (SRLE) (Garfield & Ben-Zvi, 2010). We aim to understand the perspectives of the participating teachers in this project regarding the potential and difficulties involved in the implementation of a SRLE, with particular emphasis on the big statistical ideas involved in the tasks and the used software.

The development of learning environments like the SRLE and the adoption of the new reforms proposed by the GAISE document (Franklin et al., 2007) challenge the classroom's practices. In fact, several studies show that curricular reforms are often not perceived as implying new instructional goals and a change in the adopted approaches and practices (e.g., Groth, 2008). Additionally, the use of technology, such as *TinkerPlots* software, to create different learning environments represents a new situation for teachers that may be quite challenging to them (Leavy, 2010).

CONTEXT AND METHODS

The DRP comprised two or three cycles of a sequence of task design, implementation in the classroom, analysis of results and tasks refinement. The project involved everyone in the group (11 mathematics teachers and two researchers). The teachers had diverse experiences in teaching statistics but the majority was not acquainted with the notion of statistical reasoning neither with the use of *TinkerPlots*. Data for the present study were collected through a questionnaire with 11 open-ended questions answered by the teachers at the end of the project. There was no explicit mention to SRLE in the questions but these encompass indirectly its main principles: Focus on developing central statistical ideas; use of real data, classroom activities and technology to support students' reasoning; promote classroom discourse (including argumentation) focus on statistical ideas; and use assessment to monitor students' learning. Data analysis focused on the teachers' references that have an explicit connection with those SRLE principles.

RESULTS AND DISCUSSION

Teachers recognized the utility of statistics for their students and showed to be aware of its specificity regarding the centrality of data as well as the statistical ideas and processes involved. This recognition meets the assumptions considered by Groth (2008) as necessary to an effective implementation of GAISE guidelines (Franklin et al., 2007). Contrary to theirs usual practice, the teachers did not set the use of descriptive statistics as an objective for the lessons (Leavy, 2010) but recognized the potential of the proposed tasks to work statistical ideas such as the need of data, distribution, variability and center, covariance and sampling and inference. However, they did not refer to other important aspects such as an articulated interpretation of both center and variation measures (Garfield & Ben-Zvi, 2010). They also recognize the structuring role of technology in students' work, contradicting the generalized idea of its use in teaching just to meet the program (Groth, 2008). Some teachers also mentioned the need to continue their professional development as they identified several difficulties in supporting the students regarding their reasoning. In this respect, the teachers mentioned the informal inference as a complex aspect for students since they did not consider variability when generalizing data trends and few used probabilistic language for describing the generalization. This latter difficulty may have result from a reduced attention given to this aspect, both in the lessons' preparation and in the classroom's work, particularly in the tasks' whole group discussions in the DRP.

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THE TEACHING OF MEASURES OF VARIABILITY AT SECONDARY LEVEL: EXAMINING TWO TEACHERS' PCK

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INTRODUCTION

Variability is the key component of statistical thinking to be emphasized at all school levels. Recent research shows that this concept is complex for students. To surpass their difficulties, teachers should pose tasks with diverse contexts and levels of abstraction in order to promote the learning of variability with understanding (Garfield & Ben-Zvi, 2005). The aim of this work is to analyse two teachers' pedagogical content knowledge (PCK) while they teach variability measures at secondary level. This study may contribute to better comprehend the knowledge teachers need to teach variability with understanding.

BACKGROUND

Current research shows that while students easily compute measures of variability (as range (R), interquartile range (IQR) and standard deviation (SD)) they rarely understand their meaning and have difficulties in relating these concepts with other statistical concepts and representations (Shaughnessy, 2007). Yet, teachers should help students to develop a conceptual understanding of variability (Garfield & Ben-Zvi, 2005). Teachers' practices are privileged contexts that can reveal many aspects of teachers' PCK (Ponte & Chapman, 2006). PCK is one of the two major categories of the *Mathematics knowledge for teaching* (MKT) model developed by Ball, Thames and Phelps (2008). MKT encompasses subject matter knowledge (SMK) and PCK. The latter category incorporates three components: knowledge of content and teaching (KCT); knowledge of content and students (KCS); and knowledge of curriculum (KC). Each PCK component in the MKT model considers the specificity of statistics, as noted by several authors (e.g., Shaughnessy, 2007).

METHODOLOGY AND CONTEXT

This study is part of a larger ongoing qualitative research focused on secondary teachers' PCK in statistics. This paper refers to Estela and Lia, two case studies of the wider research. Data come mainly from the observation of two 90-minute lessons on measures of variability and worksheets used in these lessons. The descriptive and interpretative analysis is guided by the PCK components, stressing KCT and KCS.

RESULTS AND DISCUSSION

With respect to KCT, both teachers introduced variability measures when the students were already working on the tasks, posing questions and guiding their work, promoting their participation. Estela's tasks asked for computing variability measures on the calculator but also for exploring their properties, and involved distribution comparisons through graph examinations using the values of mean and SD. Lia's tasks expressed the need of computing these measures for two data sets and the comparison and interpretation of the reached results. During the collective discussion of students' answers both teachers interpreted SD as a variability measure that indicates "how far are data from the mean" and considered this measure much more interesting than R or IQR. Lia often regarded the notions of mean and spread as inseparable when analysing the variability of a data set but sometimes she was not clear when pointing that the obtained mean does not mirror the observed variability on data. When comparing which of two bar diagrams had higher SD, Estela told the students to observe the deviations from the mean but sometimes she wrongly mentioned their density when stating "these differences are very big". These deviations were just referred by Lia as parcels of SD's formula. Finally, both teachers recognized the importance of using SD to compare variability of two data sets and that SD, IQR, and R serve to describe variability. Their practices also revealed the need of strengthening their SMK about variability in order to make their KCT more robust. With respect to KCS, both teachers considered that interpreting results and using variability to compare distributions are not easy activities for students due to their lack of experience. In addition, both teachers did not realize their students were using the graphing calculators in an uncritical manner and did not help them to question the results. The teachers' SMK about variability also seems to impair a stronger KCS.

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ON THE UNDERSTANDING AND COMPUTING OF PROBABILITIES THROUGH DATA IN CONTINGENCY TABLES

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INTRODUCTION

Since statistical literacy is one of the key elements for understanding the world around us and essential to the development of critical thinking, Probability and Statistics became part of school curricula from basic education to higher education. Whatever the content future teachers will teach and the education level in which it will be done, they should acquire a deeper and broader knowledge of the concepts that constitute the basis of statistical culture (Fernandes, Batanero, Correia, & Gea, 2014), in order to deal, in a confident manner, with Statistics and Probability teaching.

It is reported in literature the reduced probabilistic human intuition, and the various misconceptions associated to probability calculation. There is also evidence that these misconceptions do not disappear with the spontaneous cognitive development, so that, students arrive to higher education maintaining the same set of intuitions, in respect to probabilities, they had before (e.g., Batanero, 2013). To eliminate these misconceptions students must deal with them (Leviatan, 2002), so, we must identify students' difficulties, prior to the (single) Probability and Statistics course they will attend in higher education, in order to design strategies, to this course, which allows students to confront their misconceptions.

Much of statistical information appears in tables, so the ability to read and interpret them is essential. Another key skill of statistical literacy consists in taking appropriate decisions under situations of uncertainty (Gal, 2002). Since the information in contingency tables can be used to compute probabilities, we intend to analyze the performance of students when computing simple, joint and conditional probabilities, through data in contingency tables.

This study is a first step towards a more detailed study we intend to develop on students' statistical knowledge when arriving to higher education.

METHOD

Twenty one students, future educators and basic school teachers, participated in this study. They were posed the problem, used by Estrada and Díaz (2006, p.3), in which it is requested to determine a simple probability, a joint probability, a conditional probability and its transposed, using the information presented in a 2×2 contingency table. This task was posed at a moment prior to the attendance of the course in which the contents of Statistics and Probability will be taught.

RESULTS

The written solutions, produced by the participants, were analyzed and the responses to each question were classified taking into account the accuracy of the response, and the type of errors in the incorrect responses. In question (i), where it was required to compute the probability of a single event, $P(A)$, it was found that most of the students (90,5%) had the correct solution. In question (ii) where students should compute a joint probability, $P(A \cap B)$, 47.6% of students revealed confusion between joint probability and conditional probability. In questions (iii) and (iv), where it was asked to calculate the conditional probabilities $P(A/B)$ and $P(B/A)$, it was observed the same confusion in 23.8% and 28.6% of the responses, respectively. In question (iii), 28.6% of the responses revealed the adoption of the conditional transposed fallacy. It was also observed in (iv), although with a lower incidence (9.5%). In responses to questions (iii) and (iv) it was also observed simple probability calculation instead of conditional probabilities.

CONCLUSION

The results of this study indicate that students have many difficulties when computing probabilities from data presented in contingency tables, with high incidence of the conditional transposed fallacy and confusion between conditional probability and joint probability. These misconceptions coincide with the ones identified in other researches (e.g., Estrada & Díaz, 2006). The results suggest the need to devote special attention to these misconceptions in the course these future educators and basic school teachers will receive.

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POSTERS

STUDENTS' COVARIATIONAL REASONING: RESULTS OF A TEACHING EXPERIMENT USING TINKERPLOTS

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ABSTRACT

Covariational reasoning is referred in both national and international mathematics curriculum orientations (ME, 2001) and involves knowing how to judge and interpret a relationship between two variables (Zieffler & Garfield, 2009). Nevertheless, many studies document students' misconceptions and difficulties regarding covariation and covariational reasoning and several authors argue in favour of the use of robust graphical approaches, facilitated by appropriate use of specific technology such as *TinkerPlots*, in the teaching and learning of statistics and covariation in particular (e.g., Fitzallen, 2012). However, there are many research questions unanswered. This poster reports the results of a study aiming to analyze students' covariational reasoning in a context of a teaching experiment supported by the use of *TinkerPlots* to solve tasks involving the concept of covariation, conducted with 10th grade students. Results show that students' understanding of covariation and the assessment of correlation between variables may occur intuitively but also point to several difficulties as other studies have revealed. In this way, the results may contribute to understand the potentialities of the software and the teaching experiment in promoting students' covariational reasoning, informing teachers about the aspects needed to be focus of attention in instructional process regarding covariation. The poster begins with a presentation of the study including the aims, theoretical framework and the qualitative methodology used. It also presents a description of the teaching experiment that supports the study. The focus then is on the analysis of students' work, including several examples to document the results of the study. Finally, we draw some implications for teaching.

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BIG DATA IN GRADE 10

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ABSTRACT

Students have difficulties in representing and interpreting data (e.g., Garfield & Ben-Zvi, 2008; Lem, Onghena, Verschaffel, & van Dooren, 2013). Drawing a graph is a first step in interpreting data (Garfield & Ben-Zvi, 2008). The aim of our research was therefore to explore how students can learn to make appropriate graphs that can support valid inferences from large data sets.

An experiment was set up with 62 students in grade 10 (aged 15-16) in three Dutch pre-university classes, each in a different school. The experiment consisted of three stages: (1) teaching students to draw appropriate graphs from small data sets with Microsoft Excel, (2) teaching students to make statistical inferences, and (3) assessing students on making an appropriate graph from a large dataset. To develop statistical reasoning, questions were asked such as: 'which inference can be made from the graph you produced?'. After two lessons the students got questions about 14.000 bivariate weather data that required the drawing of an appropriate graph. The graphs produced made no sense. One reason might be that students prefer using tables instead of graphs as Lem et al. revealed (2013) and that this is connected to their choice of graphs. With a large data set this is no longer possible. It seems that students lack knowledge about how a certain graph is related to the data, e.g., that a pie chart is not suitable for bivariate data. No other data analysis is made yet. On the poster we will present some of the students' graphs and preliminary results of the experiments which include a list of do's and don'ts for the teachers and suggestions for redesigning the teaching materials. In line with Evans (2007) the use of new techniques like eye tracking is recommended for future research in order to find out the exact nature of the difficulties students encounter while constructing a graph of a large data set.

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THE CHALLENGES OF TEACHING STATISTICS TO DEAF STUDENTS

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ABSTRACT

This study aimed at identifying limits and possibilities of teaching statistics to deaf students, and at exploring pedagogical strategies to approach statistics contents with these students. Most of research publications in this area are related to curriculum topics like arithmetic or geometry (Lang & Pagliano, 2007). Therefore, our research intends to contribute to overcome the lack of research-based elements that can support the discussion of the statistics education of deaf students. Our research was qualitative in nature, had a collaborative approach, and was developed at a Portuguese elementary school that is considered as reference school for teaching deaf students. Five teachers and three researchers formed a collaborative group who met periodically to discuss how to teach statistical data handling, specifically reflecting on issues concerning the teaching and learning of statistics to deaf children and adolescents. During group discussions, several aspects identified in previous research (Borges & Nogueira, 2013) emerged: the schooling trajectory of each student; the challenges of bilingualism for students and their family; and the importance of exploring visual experiences as a resource to teaching deaf students. The group planned and developed two interventions about statistical data handling in a mixed classroom group with six deaf students and twenty listener students. After each intervention, there was a group discussion. Our findings suggested the importance of providing individualized activities and the need to use manipulative and visual materials in order to consider the deaf students' previous knowledge and experiences, meeting their specific needs, so that their understanding, performance and ultimately achievement in statistics may increase.

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INITIAL TRAINING OF TEACHERS OF MATEMATICS/STATISTICS IN SCHOOL EDUCATION

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ABSTRACT

The objective of this study is to describe and characterize the academic training of mathematics teachers in Statistics, at the basic and secondary level. It is mainly focused on the Frequency of training sessions/courses in the area of Statistics, and the need to promote teacher professional development in the area of Statistics. Data were collected through a questionnaire which was send to the 27 schools with basic and secondary education classes in the county of Évora, in the south of Portugal. The questionnaire was administered to mathematics teachers, and 70 validated responses were obtained.

STATISTICS LITERACY

Knowing Statistics is essential to citizens of modern societies: to be critical of the available information, to understand and communicate based on that information. Make all statistically competent citizens is a major goal of education Statistics and the only way to combat illiteracy statistics (Gal & Garfield, 1997; 1999; Lajoie, Jacobs & Lavigne, 1993). Shaughnessy and Bergman (1993, p.190) state, "it is crucial that researchers encourage teachers to participate in research projects because they are the key link in the chain of Statistics literacy of our students".

RESULTS

The analysis of the data suggests that attention should be paid to the importance of promote teacher professional development in the area of Statistics. There is a need to strengthen the teaching training in context in this area of knowledge, so that the difficulties experienced by students in the teaching learning process can be overcome.

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FLOW STATE EXPERIENCE TO IMPROVE THE MOTIVATION AND SUCCESS OF THE LEARNING'S MATHEMATICS EDUCATION OF YOUNG PORTUGUESE STUDENTS

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ABSTRACT

The project research to develop part of three hypothetical assumptions. The affective state as anxiety of the young students (from 14 to 16 years old), Secondary Education in Portugal, is a negative determinant of the experience of flow state (Order in consciousness to achieve of optimal concentration and satisfaction outcomes: Csikszentmihalyi, 1997), in solving of statistical ´s mathematic problems (SSMP)? The ability ´s beliefs of self-efficacy and self-confidence mediated by perceived competence and motivational states of self-determination are variables predictors of the dimensions ´s of flow state experience in SSMP? Depending on the responses of influential variables that experience an optimal ´s flow state, what intervention to be undertaken? Such research ´s assumptions will achieve through the statistical analysis of the theoretical of structural ´s equations model of causal relationship. The expected outcomes will be indicators after intervention to pursuit recommended in the achievement the work of socio-affective domain especially in the decrease the anxiety, increase of the optimal levels experience ´s flow state and a greater of self-confidence's state of students.

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GRAPHS AND TABLES IN ELEMENTARY SCHOOL: AN ANALYSIS BASED ON ACTIVITY THEORY OF ELEMENTS

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ABSTRACT

This study aimed to make an analysis the elements from didactic activities proposed by teachers in the early years of elementary school for teaching graphs and tables. The analysis was based on Activity Theory. In the theoretical approach, it was discussed about the relevance of the contents related to graphs and tables to the students' basic education and the Brazilian National Curriculum Standards. It was considered the elements of the Activity Theory and studies on Education of Statistics. We carried out a qualitative nature study in an analytical perspective. The data collection was developed using questionnaires and interviews, as well as direct observation of teachers' practices. The results indicate that the teacher's commitment to good quality teaching is necessary, but it is not enough. It is necessary that teachers know what they intend to teach, and know how to select appropriate strategies to propitiate the construction of knowledge, considering the different moments that form an activity, starting with the motivation and ending with the control. The teachers who participated in our study did not demonstrate the proper understanding of the aims of teaching about graphs and tables in the classroom, their specificities or the relationships between them. Since the motivation has an important role in the educational process, it is necessary that the teachers advance in their understanding of this element, considering its specificities, whether curricular, social or other nature. The fact that the same school has teachers who present practices which show little similarity, either regarding the content or the students, should not be considered as a whole, when we discuss teaching quality.

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THE ISLP POSTER COMPETITION: A VISUAL APPROACH OF BRINGING STATISTICAL LITERACY WORLWIDE

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ABSTRACT

The International Statistical Literacy Project (ISLP) is aimed at supporting, creating and participating in statistical literacy activities and promotion around the world. The poster competition (ISLP, 2015) is one of the main successful initiatives of the ISLP. The competition invites school students from around the world to design a statistical poster. The poster can be about any topic and reflect or illustrate usage analysis, interpretation and communication of statistics or statistical information. There are National competitions within each country involved, followed by an International competition to determine the overall winners. In 2015, these will be announced and the posters displayed at the 60th Congress of the ISI in Rio de Janeiro in Brazil during July 2015. A brief history of the competition is presented, as well as the topics, best practices, and criteria for the selection of the winning posters.

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STATISTICAL LITERACY IN PRESCHOOL EDUCATION

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ABSTRACT

In recent years, there has been in Portugal an increase in research about statistical literacy approach in educational context and about the teacher statistical knowledge necessary to promote effective statistical learning (Henriques & Oliveira, 2013). However, there is still some lack of research about the literacy development in preschool years. In this context we consider that the teachers' statistical knowledge is one core dimension that allows achieving the goal of literacy development. Such knowledge is conceived here as *Statistical Knowledge for Teaching* (Burgess, 2009).

Based on the conceptualization of Burgess (2009) about the teachers' knowledge to teach statistics, the present research aims to contribute to a better approach to statistical literacy with children from 3 to 5 years, combining the transdisciplinary approach (project approach) with teachers' statistical knowledge to promote children's effective and significant learning (Burgess, 2009; Katz & Chard, 1997).

The present research is part of a larger study, which combines interpretative qualitative methodology with study case design. Initially 11 children aged between 3 and 5 years participated in the research and was developed in a kindergarten. The information was collected during children's experiences involving investigative cycle (problem, plan, data, analysis and inference) (Burgess, 2009). Some preliminary results show that the project approach implementation related with the statistical teachers' knowledge is beneficial to promote statistical literacy and statistical reasoning in children. Are teachers conscious that their own knowledge and active learning environments are keys to the development of statistical literacy of children?

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TEACHING STATISTICS IN INDIGENOUS SCHOOLS

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ABSTRACT

Indigenous Education, as defined by Brazilian law, should be specific and differentiated from the socio-cultural expressions of each people. Authors have emphasized the importance of Statistics to support social practices of citizens (Cazorla & Santana, 2010). In teaching and learning of mathematics content, individual and cultural characteristics of the subjects should be valued in order to provide an education based on critical and reflective aspects (Skovsmose, 2000). Despite the national curriculum for Indigenous Education (Brazil, 1998) considers mathematics primordial for intercultural development of indigenous people, the aforementioned curriculum does not include statistical topics (Barbosa & Magina, 2014). This study aims to analyze the Statistic Education developed at Indigenous schools of the Xukuru of Ororubá people. The methodology is collaborative with interaction between participants and researchers at all stages of the study. The research data started being collected by means of observations, semi-structured interviews, visits to the indigenous villages and encounters with teachers. Three teachers are joining the research to develop a teaching intervention about topics of Statistics. Preliminary results show that teachers receive little guidance for the work with Statistics. In addition, usually they give priority to teaching topics related to numbers and operations in the classroom. As a result of this study, it is expected that Statistics teaching and learning processes are encouraged and combined with the reality experienced at the Indigenous villages, to make possible the legitimation and strengthening of the Xukuru of Ororubá culture.

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REFLECTION ON PRACTICES AS TEACHER EDUCATORS IN STATISTICS

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ABSTRACT

In this presentation we reflect upon our practices as teacher educators revisiting a study we developed (Barros, Martins, & Pires, 2009). The aim was (i) to identify which statistic knowledge future educators and teachers possessed when starting their training, and (ii) to assess the influence of the work undertaken in the curricular unit *Numbers and Statistics* in the progression of this knowledge. The study states that the development of the curricular unit has allowed students to deepen, change or consolidate their statistic's knowledge, mainly regarding what concerns understanding concepts. But what are the reasons for this "success"? Which aspects of our practices have contributed to a real improvement of students' learnings?

Teaching practices in Statistics

The analysis of the study has allowed us to highlight some dimensions of our practices that seem to provide more meaningful learnings.

a) *Consideration of students' needs and difficulties*, given that their identification was the starting point of the study undertaken and has determined teaching practices concerning statistics.

b) *Valuing the nature of tasks and their diversification*, performing tasks of a more closed nature as well as more open ones having been foreseen, involving the clarification and application of concepts, or completing a project.

c) *Discussion about understanding of concepts*, their comprehension having been reached through the questioning of the meaning of results.

d) *Integration and diversification of evaluation within the teaching learning process*, the different meanings of evaluation (diagnostic, formative, summative) having been valued, allowing for the reformulation of teaching strategies and practices.

e) *Empowerment of participants*, assuming (i) the teacher as a guide and moderator, proposing tasks, questioning or clarifying ideas, and (ii) the student as the builder of her knowledge, bestowing shape and meaning on her learning, within a context of active participation in the negotiation of the different meanings.

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KNOWLEDGE ABOUT ARITHMETIC MEAN: A STUDY WITH FUTURE TEACHERS

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ABSTRACT

Throughout their school years students experience different trainings and teaching processes which influence their learning of statistic concepts and procedures, taking for granted that, especially for future teachers, these concepts should be well clarified and consolidated. This analysis has as basis a study undertaken with 40 future teachers in the 1st year of the License Degree in Basic Education. The aim was (i) to identify students' statistical knowledge at the beginning of their degree, and (ii) to assess the influence of the intervention of the curricular unit *Numbers and Statistics* in the progression of this knowledge (Martins, Pires, & Barros, 2009). An investigation methodology of a qualitative kind has been followed, in which the collection of data has been conducted using a questionnaire applied at the study's beginning, participant observation during the classes and a summative test at the end. For this presentation we selected the data related to the arithmetic mean.

Knowledge of the students about the concept of mean

At the start of the curricular unit the students presented considerable difficulties in dealing with the concept of arithmetic mean. When asked to write down what they mean by "mean" almost half of them did not answer and 30% only associated it with the algorithm used for its calculation. When "calculating" the mean of a qualitative variable there were no correct answers and 43% of the students did not answer. In the case of a quantitative continuous variable, only 8% of the students performed the correct calculation and half of them did not answer. In the calculation of the mean for a quantitative discrete variable there were higher performance indexes, although 40% of the students did not answer or provided an incorrect answer.

As the curricular unit progressed, difficulties were still felt when qualitative variables (calculating the mean of the absolute frequencies was a common mistake) or continuous quantitative variables were at stake. An evolution in the understanding of the concept of arithmetic mean was noticed, with one third of the students associating it with an equilibrium value in the data.

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STATISTICAL THINKING ABOUT VARIATION IN DATA: A STUDY WITH YOUTH AND ADULT STUDENTS

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ABSTRACT

In Brazil, the teaching modality called Youth and Adult Education (EJA) aims to serve those people who did not complete their studies at the time set by the regular curriculum. Curriculum guidelines for EJA emphasize that mathematical and statistical knowledge are important for those who are returning to school, because such knowledge can help them exercise their citizenship. Statistical thinking can be defined as the ability to make appropriate use of statistical tools to solve problems, to understand the essence of the data, to make inferences, and to recognize and understand the value of statistics (Cazorla, 2002). This poster draws from a research project that will investigate aspects of students' statistical thinking who are enrolled in EJA while they respond to questions related to data variation in statistical media graphs. The data collected will be analysed from the perspective that *variability* is the characteristic of the entity that is observable, and *variation* is related to describing or measuring that characteristic (Reading & Shaughnessy, 2004). The participants will be students from an EJA group at a public school in Pernambuco, Brazil. Initially, we will apply a diagnosis instrument composed of four graph interpretation tasks. It is expected that important aspects to be analysed from students' responses will be related to their ability to identify high and low points on the graphs, as well as the identification of the intervals of increase and decrease. To further investigate the participants' interpretative processes, we will also interview some of them. We will use the SPSS software for categorizing the participants' interpretations. The qualitative analysis of our research will focus on the data from interviews. The analytical categories will be based on types of answers to the questions of graph comprehension (Friel, Curcio, & Bright, 2001).

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INTERPRETING REPRESENTED DATA: AN EARLY CHILDHOOD STUDY

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ABSTRACT

This poster will present the results of a study (Cordeiro, 2014) that aimed to understand how preschoolers represent and interpret data collected in their daily routines. The theoretical framework assumes a perspective based on the relevance of an investigative cycle in data analysis (Wild & Pfannkuch, 1999). According to Reys, Lindquist, Lambdin, Smith, and Suydam (1993), as young children develop number sense they can also develop data sense in the process of analyzing data.

The study followed a qualitative methodology, with a descriptive and interpretative character. The work was developed with the whole group of children aged 4 years 1 month to 5 years, at the beginning of data collection (January, 2014), and the second author of this poster had the dual role of educator and researcher. Data collection lasted 5 months and was gathered with the aid of video recordings and field notes focused on the children's performance.

The key results of the study showed that children transposed the data of the maps' daily routines through a diversity of correspondences, giving personal meaning to the representations. When they interpreted the data represented in graphs, they counted and compared the absolute frequencies through graphic visualization. One child understood that the total amount of the pictures of a picture graph corresponded to the number of children inside the classroom. All the work allowed children to improve their number sense and also their data sense.

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CHALLENGING INTUITION WITH THE "BERTRAND BOX" PROBLEM

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ABSTRACT

Traditional formal probability study consists on problems where only the slope of calculation is explored (Batanero, Green, & Serrano, 1998). But reducing probability study to its mathematical algorithm does not allow students neither to be prepared to solve non-routine problems nor to realize that the reasoning they are developing possibly conflicts with their intuition. Since misconceptions, associated to probability interpretation, do not disappear with spontaneous cognitive development, it is essential that students be confronted with counterintuitive situations, under the penalty of wrong intuitions remain unchanged (Leviatán, 2002). In probability teaching, the use of counterintuitive problems is a constructivist pedagogical strategy that allows approaching situations, with high incidence of errors, in a challenging and motivating way (Contreras, Batanero, Arteaga, & Cañadas, 2011). The discussion and depth analysis of these counterintuitive problems resolutions stimulates students to reflect about their thinking processes, promoting the development of abstract mathematical ability. The "Bertrand Box" problem is one of the most famous conditional probability problems where our intuition leads us to overlook or misinterpret the additional information we have. Due to its challenging nature and simplicity, it can be a starting point to challenge students' probabilistic intuition, making them aware of the importance of an adequate sample space definition. More complex variants of the problem, like the "Monty Hall" Problem, can be used to teach students how to make decisions in the face of uncertainty, in particular, by using Bayes Theorem.This work presents a theoretical study intending to show the advantages of the "Bertrand Box" and the "Monty Hall" problems as didactic tools.

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