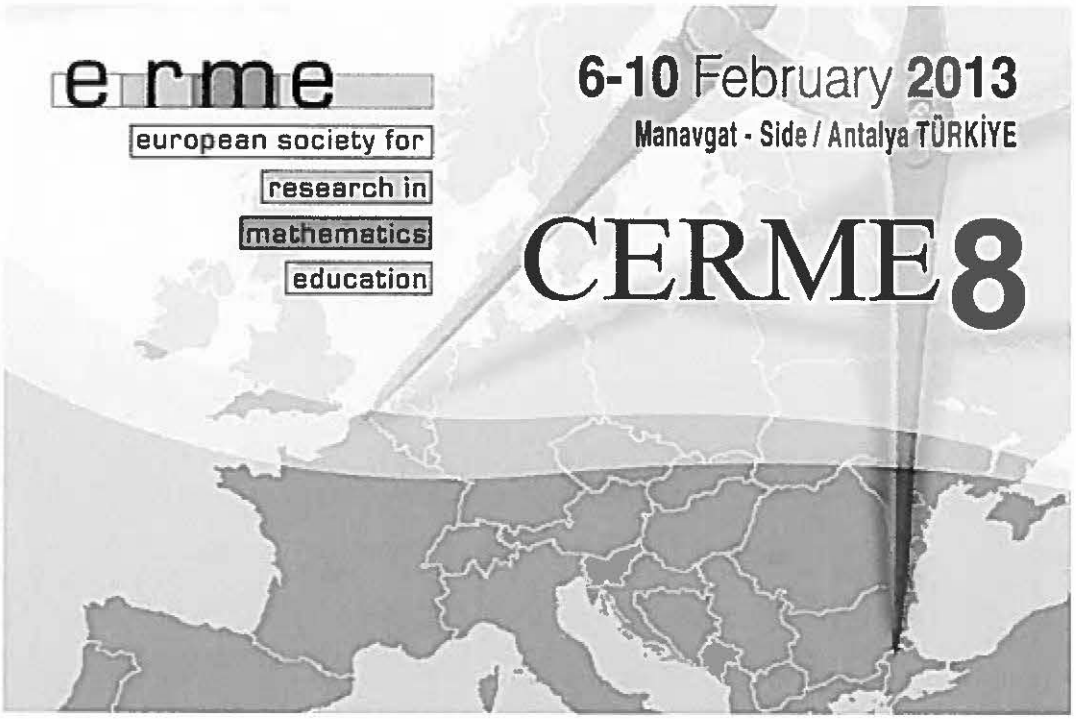

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USING APPLETS FOR TRAINING STATISTICS WITH FUTURE PRIMARY TEACHERS

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In recent years, statistics has been recognised as a basic component of citizenship education and its incorporation into school curricula in various countries confirms the importance of learning statistics. Nowadays, several resources are available online, but their uses within classes may vary depending on the use a teacher devises for them, therefore a critical analysis of these tools is needed. In this paper, we present the Applemat model for the analysis of statistical applets that might promote training statistics among primary teachers, thus emphasising their didactical purposes. During this past school year (2011/2012), an applets selection – as statistical teaching tools – was used for training primary teachers and the model was introduced in their classes. We will only present the model and the report from one of the student groups in this paper.

INTRODUCTION

Primary teachers (teachers of pupils aged from 6 to 12) must be prepared to teach statistics from the official programmes and adequately educate their students. Realistically, this would create the need to include statistics education in the curriculum of future teachers, including contents and statistical literacy, in order to allow the development of students as fully literate citizens. In our own work (Estrada, Batanero, & Fortuny, 2004), we have already found conceptual errors in elementary concepts such as mean, median and mode, outliers, standard deviation and sampling. This brought to our attention the need to rethink teaching methods. Given that statistics is one of the subjects where technologies has a major impact (Contreras, Martins, Estrada, & Batanero, 2011) we thought that we may use some of the available Internet resources – the applets – in our classes. It was thought that, due to their characteristics, they may enable us to develop a different approach to the statistical concepts. Thus, we agree with Anderson-Cook and Dorai-Raj (2003) who state: "We believe that the applets will be an easily accessible tool (...) to help students gain a better working understanding of the concepts." In our previous work (Nascimento & Martins, 2008), we have already used statistical applets with Portuguese university students, as homework, in order to make them reflect on the statistical use of their (mis)concepts and help them gain a better understanding of

how to work with them. Nevertheless, future primary teachers should have the opportunity to learn how to use applets as technological resources in statistical contexts, and attention should also be paid to the appropriate use of applets in the classroom.

In this work, we present the Applemat model version developed in the "Applemat Project" that enables future teachers to conduct a careful didactical analysis, with special emphasis on its utility in their future elementary classrooms. We also present an example of its use, devised by one group of students, as well as our own analysis of their work.

APPLETS AND TRAINING STATISTICS WITH FUTURE PRIMARY TEACHERS

In the professional development of teachers, ICT acts as a semiotic mediator that may change the epistemic configuration of the mathematics learning process (Font & Godino, 2006). However, Giménez (2004) states that teachers do not usually use these resources because they do not know their possibilities and limitations. The technological resources, namely the applets, also possess the conditions for a didactical suitability (Godino, Wilhelmi, & Bencomo (2005) later clarified by Godino, Batanero, & Font (2007) *apud* Godino, Batanero, Roa, & Wilhelmi, 2008) that defined it as the articulation of the following six types of suitability:

- Epistemic suitability: extent to which the statistical content is representative of the curricular content for a specific teaching level and whether its inclusion in the teaching is justified.
- Cognitive suitability: whether the content is adequate for the students' previous knowledge and the extent to which the instructional goals can be achieved.
- Media/resources suitability: sound use of technical tools, resources and time.
- Emotional suitability: whether the teaching/learning process takes into account the students' motivations, attitudes, affects and beliefs.
- Interactional suitability: whether the interactions between the teacher and the students and among the students themselves favour overcoming learning difficulties.
- Ecological suitability: degree to which the teaching/learning process is adapted to the social environment; possibility of establishing interdisciplinary connections.

Due to the availability of different statistical resources on the Internet, we think that teachers' training should introduce and promote the use of these ICT resources, specifically the applets, to help future teachers in recognising their value and

applicability in elementary school classrooms. More generally, Tishkovskaya and Lancaster (2012) discuss the following:

> Probably, the most common way to use information technology to enhance teaching materials in mathematics and statistics has been to add statistics applet illustrations letting students experiment with mathematical statements. Some of these illustrations are very sophisticated and valuable new elements in instruction (...) which can be accessed over the Web and used for the purpose of statistics education.

Most of the statistical applets found are devised to show contents. But Romero, Berger, Healy, and Aberson (2000) describe that "In [WISE] earlier applets (...), students interacted by pushing on-screen buttons with their cursor. In [WISE] most recent applets, students are also able to act directly on the distributions. (...)".

Anderson-Cook and Dorai-Raj (2003) also reinforce that with the applets "[t]o answer the questions, students must interact with the applet, interpret and integrate findings, and explain and apply the concepts that they have learned".

More recently DePaolo (2010) for her courses used the "four A's for applet selection": "Appropriate, Accessible, Attributes of high quality, Appealing visually".

> 1. Applets had to (...) address topics and concepts commonly taught in these courses at introductory levels. They also had to have functionality (...); 2. Applets had to be accessible to undergraduate non-majors. They had to have an intuitive interface, explanation of purpose, and brief but helpful documentation on usage (...); 3. The applets' attributes had to be of high quality. They had to have reasonable loading and running speed and be free of errors and misleading output. Applets chosen were clear in their purpose and successfully performed their intended function or demonstrated their intended concept; 4. Applets had to be appealing visually. They had to have effective (though not necessarily fancy) graphics and produce output that is easy to understand and interpret.

In our view this "four A's applet selection" criterion (4AASC) adapted to the statistics curriculum of the prospective teachers or even of the elementary school one is a valuable guideline for choosing a "good" applet. Connecting with the didactical suitability components, the epistemic or mathematical suitability denotes the future teacher choice of a "good applet" using 4AASC adapted to the statistical contents of those elementary grades and also to the class kind of task for the pupils (either introductory, reinforcement or even test tasks). On the other hand, when the future teacher includes its adequate didactical use in the classroom work for that task we are referring to the cognitive suitability. "[T]eachers who are able to use today's technology in the classroom will be prepared to learn and utilise tomorrow's technology" (Powers & Blubaugh, 2005) stresses media/resources suitability. The emotional suitability, as pointed out by Díaz and de la Fuente (2005), determines that

using applets for teaching statistics and probability increases students' motivation for the subject because they present the concepts in a more attractive way and the interactional suitability also enables them to play a more active role in their own learning. Taking these aspects into account, it is important that the teacher considers how to use these resources effectively – emphasising epistemic and cognitive suitability. So, using the applets, as well as doing their didactical analysis, may help future teachers in the statistics learning process – ecological suitability. In prospective teachers' training, the methodological techniques should be implemented to give them the incentive to incorporate different types of practices into their training. In this paper we will only present the Applemat model and the analysis of one report from a group of students where we articulate the criteria for the applet's selection, its assessment and its adequacy for didactical analysis based on the six types of didactical suitability presented earlier. Further work with the other students' reports will be presented in the near future.

METHOD

The first approach to the model involved an answer to the question of how future teachers should conduct the didactical analysis of the applets – as a teaching tool. This approach was developed within the "Applemat Project" (2006–2007), as part of the "Teacher Innovation Group" (TIG). The TIG group was coordinated by J. Giménez from 2006 to 2007 and received funds from the Catalonia government. The TIG working group had experienced university teachers from different areas – analysis, geometry, statistics, etc. – that felt the need to improve the ICT use in the university classrooms of prospective teachers. The model was an attempt to standardise the didactical analysis for all the scientific areas that the university students are required to know. In the two years, TIG elements viewed the components described as conditions for didactic suitability and developed the model with three sections. In the first section, the resource is described and its possible use in elementary classes is discussed. In this section, the weight of the six components is higher regarding the epistemic suitability and the cognitive suitability. In the second section, the applet is devised as a teacher training tool in order to manage and use it during teachers' training university classes, so the weight of the six components is distributed amongst them. Finally, in the third section, the possible improvements and extensions of the applet are proposed and the weight of the six components is also distributed amongst them. The Applemat model's details are presented in Table 1.

Section 1. Applet analysis and possibilities	Section 2. Management and use in class for teachers'	Section 3. Applet improvements and

	training	extensions
1.1. Applet description 1.2. Analysis and uses in primary class a. How may we use it in the class? b. Previous knowledge and contents required c. Limitations d. Techniques of use e. Other contents that may be incorporated into the class	2.1. Goals 2.2. Developed professional skills 2.3. Transfer of learning a. Initial problematic situation b. Previous knowledge reinforcement c. Using the applet	3.1. Applet improvements or extensions 3.2. Other learning possibilities 3.3. Alternative materials' advantages and disadvantages

Table 1: Applemat model details

In our research, this model was used in the Primary Teachers degree classes (3rd year) in a Spanish university 1st cycle (Bologna Degree) attending the Probability and Statistics Curricular Unit (course) in the school year of 2011/2012 (six credits from the European Credits Transfer and accumulation System, ECTS). In this course syllabus, besides the introduction to data analysis, probability and statistics contents, the didactics of probability and statistics are also present. The 123 students enrolled in this course were divided into two classes (58 and 65 students) and they presented a total of 32 works that were carried out in groups of three or four students.

Students using the Applemat model were not familiar with the didactical analysis of applets or any other ICT resources. Thus, the university teacher presented and subsequently practised the central tendency measures with the students, before using the Illuminations mean and median applet (retrieved from the NCTM Illuminations http://illuminations.nctm.org/LessonDetail.aspx?ID=L452) as the learning objectives, materials, the instructional plan with the task, the discussion and the questions for students were already available on the site. The students' task was then to search the Internet for different applets and select one to analyse using the Applemat model (Table 1). The basis for choosing their own applet must be one or both of the subjects: their course (at the university) syllabus topics, or the elementary curriculum topics on data analysis or probability. Once they had selected it, students worked as a team to apply the Applemat model. Finally, they had to present a written report of their work. This analysis was delivered at the end of the curricular unit and was assessed by the teacher, representing 25% of the final grade.

The assessment criteria are important in any course both for teachers and students, so the university teacher designed a first guideline evaluation document that included the 4AASC. The presented assessment criteria considered four groups: following instructions, presentation and style, statistical content and knowledge for elementary curriculum, and thinking and analysis. Firstly, following the instructions group

assessed the adequacy of the applet choice for the goals of this course, the prescribed parameters of the task, and those of the model, and of the learned statistical topics. Secondly, for the presentation and style criteria the graded items were the clarity of the report text (including spelling, grammar and punctuation), its visual presentation (also considering its creativity and illustrations that aid understanding, integrated within the text), and finally the key aspects of its precision and rigour using statistical terms, expressions and concepts and their meanings. Thirdly, in what concerns the statistical content and knowledge of the Applemat model, the reports were analysed considering the depth and awareness of the statistical and didactical knowledge focusing on: applet description; possible classroom uses and applications; previous and actual statistical knowledge contents required; techniques of the applet uses and their limitations; other possible applications, improvements or extensions; and alternative materials. Finally, the thinking and didactical analysis of the reports assessed the integration of a critical perspective along the report, as well as the significance of the proposed recommendations.

Lastly, this school year of 2011/2012 was the first year in which the guideline evaluation document was used. As written by Biehler (2005) in the students' project reports scope we consider having a similar situation with our guideline: "Our work on analysing students' projects, developing a new project guide, and developing an assessment scheme is still in progress."

AN EXAMPLE USING THE MODEL

From the 32 groups of future primary teachers, a didactical analysis using the model (Table 1) was chosen and is presented here as an example of its implementation. This work was selected based on the applet's characteristics – available for download to our own computers – and its good grade. We only translated the main lines of the students' analysis regarding the "Tables and Statistical Graphs" Spanish applet report (http://www.edu.xunta.es/espazoAbalar/espazo/repositorio/cont/tablas-y-graficos-estadisticos), and now we present it.

Section 1. Applet analysis and possibilities

1.1. Applet description: This applet is an attractive visual resource that allows students to use elementary statistical techniques to obtain information about children's daily contexts including data representations, graphics and numbers, and also allows for critical reflection of the results. This applet also helps to describe, extract and interpret the information presented in the tables or in the graphs (...) which is based on daily problem solving. Furthermore, this applet has six folders. Each one is about one statistical chapter and we enter it by selecting one of the six persons (characters). It also contains a glossary and a guide for teachers. During its use, there is a voice that explains the activities and these explanations are also written in the applet window. 1.2. Analysis and uses in elementary classes: a. How may we use it in the class? (...) this applet has six folders and each one is about one statistical

chapter: data techniques of collecting and classification (4 activities); building absolute frequency tables (4 activities) and relative frequency tables (3 activities); frequency polygons (4 activities); pie charts (5 activities); reading and interpretation of statistical concepts; b. Previous knowledge and contents required: reading double entry tables; reading coordinate axes; circular sector, including the concept of central angle measure; per cent, and the per cent value of a fraction; ability to use a calculator; c. Limitations: the applet does not have the possibility of error because if the answer is wrong, the student cannot continue the activity; the activities are not connected to each other; there is no way to change data; not all activities are for all elementary school levels; d. Techniques of use: choosing a correct answer; putting data into frequency tables; checking data; arithmetic operations like addition and subtraction; reading graphs; comparing graphs or data; completing a frequency table taking into account the added data; building a table from graph data; writing a legend; writing the title; e. Other contents that may be incorporated into the class: computations and measure.

Section 2. Management and use of class for teachers' training

2.1. Goals: explore the use of ICT in mathematics learning; learning to learn; development of manipulation and visualisation as a didactical procedure; (...) understanding of frequency tables; potentiate the study and the understanding of data graphs; development of the study and its interpretations using data; learn how to teach. 2.2. Developed professional skills: analysis skills; self-learning ability; critical thinking before the methodological procedures of learning; promote cooperative work; using, applying and creating manipulative resources to learn (...). 2.3. Transfer of learning: a. Initial problematic situation: if the user chooses chapter six – the baby with the bear – data information may have different graphic types, graphics may be compared or problems with statistical graphics may be proposed. (...) a chain of events is triggered when each character is chosen; b. Previous knowledge reinforcement: data techniques of collecting and classification; building absolute and relative frequency tables; frequency polygons; pie charts; reading and interpretation of statistical graphics; c. Using the applet: using interactive learning applets as a didactical resource; different ways of representing data; valuing different ways of data presentation, reading and interpretation.

Section 3. Applet improvements and extensions

3.1. Applet improvements or extensions: if the answer is wrong, the student cannot continue the activity, so this option should be changed and the student should have the opportunity to know what their mistake was. Since the activity folders are not connected to each other, some kind of connection may be implemented in the future; devise a way to allow data changing and provide a dynamic update of the graphic. 3.2. Other learning possibilities: in these students' work, another activity folder was proposed in order to connect all the contents in the other six folders already available; create activities in the folders in order to include some with percentages; include

pictograms; some more activities to reinforce students' training. 3.3. Alternative materials' advantages and disadvantages: 1. Building tables and graphics with a worksheet; Advantages: All computers have worksheets. Enables the discovery of how this survey tool helps students' use of statistics in their daily lives as well as in other subjects. Allows building any graphics type; Disadvantages: Activities must be guided by the teacher, at least in the beginning, since the software is not particularly accessible to younger students. The difficulty levels of learning are not easily controlled by the teacher; 2. Motivating element, Advantages: Activities are based on contents and are linked between them; Disadvantages: If the connection between pages is lost it is impossible to finish the activity.

From our analysis of these students' work, we think that the group had a good report since the students followed instructions, i.e., the proposed Applemat model and the main 4AASC. In our view, Section 1 of the model report shows that the students used/explored the applet in order to understand it fully. The report had a simple and clear presentation and style. The statistical content and knowledge for the elementary curriculum was adequate and without wrong definitions. Lastly, the item about thinking and analysis was good in this report as the group was able to present a first critical opinion about this applet, as well as considerations that allow a better understanding of its use in the classroom. Sections 2.1 and 2.2 were weaker, nevertheless the students attempted to offer a glimpse of the goals and a lighter view of the professional development. The topics of subsection 2.3 were good in describing the transfer of learning. In relation to Section 3 of the model, the report was good as it detailed improvements and extensions of the applet. Overall, the group report was good using the Applemat model for the applet's didactical analysis.

FINAL REMARKS

The availability of Internet resources enables statistical learning to be accomplished in a different way, using these resources as dynamic promoters. In this work, we reviewed some of the perspectives concerning the applet's uses and we presented a model that provides guidelines for the applet's didactical analysis, within future primary teachers' training scope. In our own view, applets – as technological resources – and the Applemat model with its three sections articulated the six components for the didactic suitability, as defined by Godino et al. (2005) and later clarified by Godino et al. (2007, *apud* Godino et al., 2008). Concerning the epistemic or mathematical suitability, the Applemat model described was suitable for the future teachers to use/explore the statistical concepts, for instance, in the tables and graphs presented in the students' example. They made use of their statistical knowledge choosing and exploring the chosen applet with this model. In general, the "statistical content in (...) primary school level (...) can be introduced and justified through [an

applet task] understood by the student" (Godino et al., 2008). With regards to the cognitive suitability, the Applemat model showed potentialities since the future teachers had to report the applet's adequate didactical use in the classroom. Again we stress that using the applets, as well as doing their didactical analysis, may help future teachers in the statistics learning process – ecological suitability. According to the media/resources suitability, a single computer and an Internet connection for each group of students will be enough. Also, students may suggest different alternative materials as they did in Section 3 of their report. Since the interactional suitability mainly depends on how the teacher organises their work in the classroom, the future teachers were required to work in groups in order to encourage conflicts and verbalise their occurrence. A drawback of this year use of the Applemat model was the shortness of time to promote the class discussions. In this case, the emotional suitability was viewed as the students' involvement (interest, motivation ...) in the study process by means of the Applemat model use. We consider that this was the most appropriate one and was the one that triggered the present work. Using the Applemat model, future primary teachers were able to value applets as a didactical resource based on their own observations and manipulations – with the detail given through the presented report of the group. To conclude, this school year of 2011/2012 was the first year in which the guideline evaluation document was used, so the assessment guideline is still in an improving stage. We are convinced that the university students' work should continue to be carried out in small groups to promote opinions and discussion that will potentiate their critical thinking, which is essential in the building of an "almost real" didactical proposal. Following our analysis of the work done we propose a four-step approach in the future teachers' classroom. Firstly, we will present them this year's work so they will have a basis from which to explore the model themselves. Secondly, they will do this year's work. Thirdly, after receiving the teacher's feedback, the students will discuss their own work in order to clarify their own concepts. Finally, in order to enhance the future teachers' didactical training for the elementary levels, they will prepare and simulate – for instance, for their colleagues – the didactical sequence in order to test it and improve it. With this approach, the applets will act as the semiotic mediator in order to change the statistics learning process, as already underlined by Font and Godino (2006). Although in the teacher analysis of this school year 2011/2012 almost all the reports got good grades, we intend to further develop their analysis in a qualitative and comparative way, detailing the differences between them, as well as any written conceptual (of probability or statistics) errors made. Finally, similarly to Contreras et al. (2011), we think that to promote an improvement of the statistical learning in

elementary schools, teachers should take all these resources into consideration, particularly the applets. Therefore, these resources should be implemented in future teachers' training in a similar way to the methods outlined in the current paper.

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