Remarks on the Vietoris Sequence and Corresponding Convolution Formulas

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Abstract. In this paper we consider the so-called Vietoris sequence, a sequence of rational numbers of the form $c_k = \frac{1}{2^k}\binom{k}{\lfloor \frac{k}{2} \rfloor}$, $k = 0, 1, \ldots$. This sequence plays an important role in many applications and has received a lot of attention over the years. In this work we present the main properties of the Vietoris sequence, having in mind its role in the context of hypercomplex function theory. Properties and patterns of the convolution triangles associated with $(c_k)_k$ are also presented.

Keywords: Sequences · Central binomial coefficients · Convolution triangles · Clifford algebra

1 The Vietoris Sequence

For our purpose here, we define the Vietoris sequence $(c_k)_k$ in terms of the "complete central binomial coefficient" as

$$c_k := \frac{1}{2^k}\binom{k}{\lfloor \frac{k}{2} \rfloor}, \; k = 0, 1, \ldots, \tag{1}$$

where $\lfloor \cdot \rfloor$ is the floor function. The first terms of this sequence are

$$1, \frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{3}{8}, \frac{5}{16}, \frac{5}{16}, \frac{35}{128}, \frac{35}{128}, \ldots .$$

Some years ago, authors of this paper noticed that the sequence (1) appeared in the construction of sequences of multivariate generalized Appell polynomials [11,22]. Since then, several studies on this sequence have been carried out (see e.g. [6,8–10] and the references therein) and the importance of this sequence in hypercomplex context is unquestionable nowadays. For this reason, we thought it would be interesting to collect the properties that have been obtained over the years, presenting them in a unifying way.

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