Starting with the Differential: Representation of Monogenic Functions by Polynomials of Non-monogenic Variables

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Abstract. This paper deals with different power series expansions of generalized holomorphic (monogenic) functions in the setting of Clifford Analysis. Our main concern are generalized Appell polynomials as a special class of monogenic polynomials which have been introduced in 2006 by two of the authors using several monogenic hypercomplex variables. We clarify the reasons why a particular pair of non-monogenic variables allows to obtain a power series expansion by those generalized Appell polynomials. The approach is based on the differential of a function. Some other monogenic polynomials as well as applications are mentioned.

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Introduction

Higher-dimensional analysis relying on non-commutative Clifford algebra tools is frequently called *Clifford Analysis*. Besides work of Weierstrass at the end of the 19th century about functions of two complex variables or others about functions of quaternions, the time for such methods matured only at the end of the second decade of the 20th century. The incubation time ended mainly due to research done by R. Fueter [14] from the 30ies on. Almost during 20 years he developed with his pupils a theory of quaternionic functions of a quaternion variable as the theory of general-ized Cauchy-Riemann or Dirac differential equations. Naturally, solutions of those systems have been considered as (hypercomplex) *regular or generalized holomorphic functions.* The book [6] favored the equivalent name *monogenic functions* that we also use. Two decades after Fueter, a great part of Fueter's work was renewed or actualized by following Riemann's approach through partial differential equations, a long time thought as the only one reasonable. Nevertheless, it led very quickly to a generalized function theory as refinement of Harmonic Analysis. But this type of function theory heavily relies on representation theoretic and algebraic tools, and much less on instruments from classical complex function theory. For a long time applications, mainly based on polynomials, to number theory (the main motivation for Fueter himself) or methods related to combinatorial questions were almost neglected. This drawback substantially restricted the class of functions useful for a treatment in more analytically oriented research. The main reason for such a situation was that *powers of one hypercomplex variable* (and corresponding polynomials or power series) *do not belong to the set of monogenic functions* in the sense of Fueter. In the 90ies the papers [16, 17, 18] contributed to a radical change of this perspective, describing the same class of regular functions by using several hypercomplex variables (Fueter variables) as a new starting point for a hypercomplex function theory. The papers also clarified the fact that differentiability as property of local linear approximation and derivability (the existence of a hypercomplex derivative) are, contrary to the complex case $n = 1$, dual and have to be considered for $n > 1$ in hypercomplex dimension one resp. in co-dimension one of $\mathbb{R}^{n+1}$ [20]. More about this approach and its applications the reader can find in the recent paper [2].

Starting with the differential we follow now the approach in [16, 17, 18] and consider monogenic functions and their power series representation in general as well as in terms of generalized Appell polynomials [11]. They can be found in 3D-quasi-conformal mappings [19], new analytic methods in elasticity [4, 5], or the construction of generalized Hermite, Laguerre [7], Chebyshev [8], as well as Bernoulli, Euler and other polynomials [1].

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