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A Sturm-Liouville equation on the crossroads of continuous and discrete hypercomplex analysis

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The paper studies discrete structural properties of polynomials that play an important role in the theory of spherical harmonics in any dimensions. These polynomials have their origin in the research on problems of harmonic analysis by means of generalized holomorphic (monogenic) functions of hypercomplex analysis. The Sturm-Liouville equation that occurs in this context supplements the knowledge about generalized Vietoris number sequences $\mathcal{V}_n$, first encountered as a special sequence (corresponding to $n = 2$) by Vietoris in connection with positivity of trigonometric sums. Using methods of the calculus of holonomic differential equations, we obtain a general recurrence relation for $\mathcal{V}_n$, and we derive an exponential generating function of $\mathcal{V}_n$ expressed by Kummer's confluent hypergeometric function.

KEYWORDS
Clifford algebra, hypercomplex analysis, Sturm-Liouville equation, Vietoris' numbers

MSC CLASSIFICATION
30G35; 11B83; 05A15; 34B24

1 | STARTING THE JOURNEY—SOME HISTORICAL REMARKS

> *The experience of past centuries shows that the development of mathematics was due not to technical progress (consuming most of the efforts of mathematicians at any given moment), but rather to discoveries of unexpected interrelations between different domains (which were made possible by these efforts).*
>
> Vladimir I. Arnold
> (in: *Polymathematics: Is Mathematics a Single Science or a Set of Arts*)

Vladimir I. Arnold, one of the most influential mathematicians of the recent past, is well known for having contributed with unconventional reasoning to an astounding number of different mathematical disciplines. This is marvelously visible in his contribution[1] to the volume of the International Mathematical Union celebrating the year 2000 as the Year of Mathematics. In the beginning of his paper, he mentioned that

> “According to J. J. Sylvester (1876) a mathematical idea should not be petrified in a formal axiomatic setting, but should be considered instead as flowing as a river.”

Dedicated to Klaus Guerlebeck on the occasion of his 65th birthday.

