Non-symmetric Number Triangles Arising from Hypercomplex Function Theory in $\mathbb{R}^{n+1}$

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Abstract. The paper is focused on *intrinsic properties* of a one-parameter family of non-symmetric number triangles $\mathcal{T}(n)$, $n \geq 2$, which arises in the construction of *hyperholomorphic Appell polynomials*.

Keywords: Non-symmetric Pascal triangle · Clifford algebra · Recurrence relation

1 Introduction

A one-parameter family of non-symmetric Pascal triangles was considered in [8] and a set of its basic properties was proved. Such family arises from studies on generalized Appell polynomials in the framework of Hypercomplex Function Theory in $\mathbb{R}^{n+1}, n \geq 1$, (cf. [7]). If $n \geq 2$, it is given by the infinite triangular array, $\mathcal{T}(n)$, of rational numbers

$$T_s^k(n) = \binom{k}{s} \frac{(\frac{n+1}{2})_{k-s}(\frac{n-1}{2})_s}{(n)_k},\ k = 1, 2, \ldots, ; s = 0, 1, \ldots, k, \tag{1}$$

where $(a)_r := a(a+1)\ldots(a+r-1)$, for any integer $r \geq 1$, is the Pochhammer symbol with $(a)_0 := 1, a \geq 0$. If $n = 1$, then the triangle degenerates to a unique column because $T_0^k(1) \equiv 1$ and, as usual $T_s^k(1) := 0,\ s > 0$.

The non-symmetric structure of this triangle $\mathcal{T}(n)$ is a consequence of the peculiarities of a non-commutative Clifford algebra $\mathcal{C}\ell_{0,n}$ frequently used in problems of higher dimensional Harmonic Analysis, like the solution of spinor systems as n-dimensional generalization of Dirac equations and their application in Quantum Mechanics and Quantum-Field Theory [6].

Hypercomplex Function Theory in $\mathbb{R}^{n+1}$ is a natural generalization of the classical function theory of one complex variable in the framework of Clifford Algebras. The case $n > 1$ extends the complex case to paravector valued functions

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